JOINT INSTITUTE FOR NUCLEAR RESEARCH

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FINAL REPORT ON THE START PROGRAMME

Multiparton production and Decay of Φ Meson

Supervisor:
Professor: Elena Kokoulina

Student:
Yeirys Caballero Durán, Cuba
Higher Institute of Technology and Applied Sciences

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Abstract

This study explores Quantum Chromodynamics (QCD) jet evolution, focusing on gluon fission, quark bremsstrahlung, and quark-pair creation processes. Using generating functions, we derive probabilistic descriptions of gluon and quark jets, analyzing multiplicity distributions under simplified conditions. The Gluon Dominance Model (GDM) is applied to describe hadron production in e^+e^- annihilation and proton-proton collisions, with multiplicity fits using negative binomial and Polya-Eggenberger distributions. Simulations in Pythia validate the model, showing agreement with experimental cuts. Additionally, the invariant mass spectrum of phi mesons (decaying to K^+K^-) is analyzed via relativistic Breit-Wigner fits, separating signal from background.

1 Introduction

1.1 Quark and Gluon jets

Jets are robust tools for studying short-distance collisions involving quarks and gluons. With a suitable jet definition, one can connect jet measurements made on clusters of hadrons to perturbative calculations made on clusters of partons. More ambitiously, one can try to tag jets with a suitably defined flavor label, thereby enhancing the fraction of quark-tagged jets over gluon-tagged jets. This is relevant for searches for physics beyond the standard model, where quarks often dominate signals of interest while gluons dominate the corresponding backgrounds. A wide variety of quark/gluon discriminants have been proposed, and there is a growing catalog of quark/gluon studies. Due to the complicated radiative showering and fundamentally non-perturbative hadronization that occurs in the course of jets emerging from partons, there is no unambiguous definition of “quark” or “gluon” jets at the hadron level. Despite this challenge, the importance of a clear, well-defined, and practical definition of quark and gluon jets at modern colliders cannot be overstated.[1]
Definition of a quark and a gluon jet. A phase space region (as defined by an unambiguous hadronic fiducial cross-section measurement) that yields an enriched sample of quarks (as interpreted by some suitable, though fundamentally ambiguous criterion). The Large Electron-Positron Collider (LEP) measured the ratio of the number of particles in gluon vs. quark jets. The average multiplicity of any type of particle, along with its variance, is given by the semiclassical approximation $\frac{<N_g>}{<N_q>} = \frac{C_A}{C_f}$ and $\frac{<\sigma_g^2>}{<\sigma_q^2>} = \frac{C_A}{C_f}$. Where C_f and C_A are the color charges for gluons and quarks, respectively, and $\frac{C_A}{C_f} = \frac{9}{4}$. [8]
An intuitive explanation for these results is that a quark jet is dominated by the first gluon emission, at which point it continues to shower like a gluon jet. Since gluon jets have more particles for a given energy, they will have correspondingly fewer hard particles. In cases where QCD estimates do not agree with full simulation or with data, the reason is often attributed to energy conservation not being taken into account in each splitting. Since Monte Carlo showers enforce this energy conservation, they often have better agreement with data than with the analytic estimates. Multiplicities have been calculated, including energy-momentum conservation, at N3LO. At LEP I energies, the result was $\frac{<N_g>}{<N_q>} \approx 1.7$. The charged particle multiplicity in light quark jets of average energy 45.6 GeV and gluon jets of 41.8 GeV was studied. Agreement in the moments (mean, width, skewness, kurtosis) of the particle-count distributions was found to agree with the Monte Carlo event generators and with analytic predictions. Sub-jet multiplicities were also examined at LEP for various subject sizes. Extremely small sub-jets ($K_T = 0.1GeV$) approach the limit of particles, and therefore probed hadronization. However, larger sub-jets ($K_T = 5GeV$) probed the better modeled, perturbative physics and gave the largest ratio between quark and gluon sub-jet multiplicities. For the first study cited, the average energy of the quark jets was 32 GeV, while that for gluon jets was 28 GeV.[3]

1.2 Branching Markov Processes

A stochastic process is the counterpart of a deterministic process. Even if the initial condition is known, there are many possibilities of how the process might go, described by probability distributions. More formally, a stochastic process is a collection of random variables $\{X(t), t \in T\}$ defined on a common probability space indexed by the index set T, which describes the evolution of some system. One of the basic types of stochastic processes is a Markov process. The Markov process has the property that conditional on the history up to the present, the probabilistic structure of the future does not depend on the whole history but only on the present. The future is, thus, conditionally independent of the past. The original purpose of branching processes was to serve as a mathematical model of a population in which each individual in a generation produces some random number of individuals in generation, according, in the simplest case, to a fixed probability distribution that does not vary from individual to individual.[5]

1.3 Generating Function

The probability generating function of a discrete random variable is a power series representation (the generating function) of the probability mass function of the random variable. Probability generating functions are often employed for their succinct description of the sequence of probabilities Pr(X = i) in the probability mass function for a random variable X, and to make available the well-developed theory of power series with nonnegative coefficients. A generating function is a different way of writing a sequence of numbers. The interest of this notation is that certain natural operations on generating functions lead to powerful methods for dealing with recurrences on the coefficients. Definition: Let $(a_n)_{n>0}$ be a sequence of numbers. The generating function associated with this sequence is the series.[14]

$$G(x) = \sum_{n>=0} a_n x^n \tag{1}$$

where a_n is the number of objects of size n in the class. If $x = (x_1, ..., x_d) = X$ is a discrete random variable taking values in the d-dimensional non-negative, the probability generating function of X is defined as:

$$G(z) = G(z_1, ..., z_d) = \sum_{x_1,...,x_d}^{\infty} p(x_1, ..., x_d) z^{x_1} .. z^{x_d} \tag{2}$$

The expectation of X is

$$< x >= \frac{\partial G(z)}{\partial z}|_{z=1} \tag{3}$$

The variance of X is

$$D^2 = \frac{\partial G(z)^2}{\partial^2 z}|_{z=1} - \frac{\partial G(z)}{\partial z}|_{z=1} - (\frac{\partial G(z)}{\partial z}|_{z=1})^2 \tag{4}$$

And its second correlative moment is

$$f_2 = D^2 - < x > \tag{5}$$

1.4 Two Stage Model

It was interpreted that the natural QCD evolution parameter is the thickness of the QCD jets.

$$Y = \frac{1}{2\pi b} log(1 + \alpha log(\frac{Q^2}{\mu^2}) \tag{6}$$

Three main elementary processes contribute with different weights to the overall Quark or gluon distributions inside QCD jets:
1. gluon fission: $g \to g + g$
2. quark bremsstrahlung: $q \to q + g$
3. quark pair creation: $g \to q + \bar{q}$

Let $A\Delta Y$ be the probability that a gluon in the infinitesimal interval ΔY will convert into two gluons, $\tilde{A}\Delta Y$ the probability that a quark will radiate a gluon, the quark continuing on its way, and $B\Delta Y$ the probability that a quark-anti-quark pair will be created from a gluon. A, $\tilde{A}$, and B are assumed to be Y-independent constants, and each parton acts independently from the others. The generating functions for a gluon jet and a quark jet will be, respectively, [4]

$$G(u_g, u_q; Y) = \sum_{n_g, n_q = 0}^{\infty} P_{1,0,n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} \tag{7}$$

$$Q(u_g, u_q; Y) = \sum_{n_g, n_q = 0}^{\infty} P_{0,1,n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} \tag{8}$$

Where $P_{m_g,m_q,n_g,n_q}(Y)$ is the probability that m_g gluons and m_q quarks will be transformed into n_g gluons and n_q quarks.
The probability for a gluon or a quark to produce, in the interval $(Y + \Delta Y)$, n_g gluons and n_q quarks through processes 1-3 under the requirement of probability conservation is, for a gluon jet:

$$P_{1,0,n_g,n_q;}(Y) = [1 - \tilde{A} n_q \Delta Y - A n_g \Delta Y - B n_g \Delta Y] P_{1,0,n_g,n_q;}(Y) + \tilde{A} n_q \Delta Y P_{1,0,n_g-1,n_q}(Y) + A(n_g-1)\Delta Y P_{1,0,n} \tag{9}$$

And for a quark jet we only need to write $P_{0,1,n_g,n_q}$ instead of $P_{1,0,n_g,n_q}$ in every case .

The evolution of the generation function for quarks and gluons can be obtained from the following system of equations:

$$\frac{\partial G}{\partial Y} = AG^2 - AG - BG \tag{10}$$

$$\frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG \tag{11}$$

With initial conditions:

$$G(u_g, u_q, 0) = u_g \tag{12}$$

$$Q(u_g, u_q, 0) = u_q \tag{13}$$

1.5 Solutions in particular cases

Finding the explicit solution of the cross-section is not easy. However, approximate solutions can be obtained for particular cases, which can help us understand the general problem. The case to consider is $B = 0, A \neq \tilde{A} \neq 0$, meaning that no pair creation is allowed.[4] The evolution of the generation function is:

$$\frac{\partial G}{\partial Y} = AG^2 - AG \tag{14}$$

$$\frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG \tag{15}$$

With initial conditions:

$$G(u_g, u_q, 0) = u_g \tag{16}$$

$$Q(u_g, u_q, 0) = u_q \tag{17}$$

Then, the probability for a gluon to produce n g gluons is:

$$P_{1,0,n_g,0}(Y) = e^{-AY}(1 - e^{AY})^{n_g - 1} \tag{18}$$

And the mean number of gluons in the jet is:

$$< n_g >= e^{AY} \tag{19}$$

The corresponding generating function is:

$$G = \sum_{n_g=0}^{\infty} u_g n_g P_{1,0,n_g,0}(Y) = \frac{u_g e^{-aY}}{1 - u_g(1 - e{-}AY)} \tag{20}$$

$$D^2 =< n_g >^2 - < n_g^2 >= e^{AY}(e^{AY} - 1) \tag{21}$$

The probability for a quark to produce n_q quarks is:

$$P_{0,1,n_g,1} = \frac{\mu(\mu+1)...(\mu+n_g-1)}{n_g} e^{-AY}(1 - e^{-AY})^{n_g} \tag{22}$$

$$\mu = \frac{\tilde{A}}{A} \tag{23}$$

$$< n_g >= \mu(eAY - 1) \tag{24}$$

The corresponding generating function is:

$$Q = \sum_{n_g=0}^{\infty} u_g^{n_g} u_q^{n_q} P_{0,1,n_g,1} = u_g(\frac{e^{-AY}}{1 - u_g(1 - e^{-AY})})^{\mu} \tag{25}$$

$$D^2 =< n_g >^2 - < n_g^2 >= \mu e^{AY}(e^{AY} - 1) \tag{26}$$

1.6 Gluon Dominance Model

Multiple production(MD) of secondary particles was studied in many experiments using the Gluon Dominance Model(GDM). This model is a convolution of two stages. The first stage, called the quark-gluon cascade, is described by the differential-difference equations as a Markov branching process. Two processes are allowed, bremsstrahlung of a gluon by a quark and gluon fision, formation of a quark pair is not possible according to QCD estimates [4].

The second stage, hadronization, it's described using a phenomenological scheme based on the behavior of the second correlation moment f [13] The second correlation moment calculated for the MD in quark and gluon jets is always positive at any energy. Considering that at low energies the cascade is poorly developed and hadronization predominates, the binomial distribution with negative f_2 was used to describe the hadronic probabilities

$$P_m^h = C_{N_p}^m (\frac{\bar{n_p}^h}{N_p})^n (1 - \frac{\bar{n_p}^h}{N_p})^{N_p - n} \tag{27}$$

Where the index p corresponds to q or g in the case of quark jet and gluon jet, respectively, $C_m^{N_p}$ is the binomial coefficient, $\bar{n_p}^h$ is the average possible number of hadrons formed at the hadronization stage, and N_p is the maximum number of hadrons.

1.7 e^+ e^- annihilation

The GDM has been proposed to describe the MD of secondary hadrons in e^+ e^- annihilation. The generation functions can be used to describe multiplicity distribution(MD)[9][4]. By definition, the generation function is a convolution of the MD

$$Q(s, z) = \sum_n P_n z^n \tag{28}$$

Where s is the square of the initial energy.

$$P_n = \frac{1}{n!} \frac{\partial^n}{\partial z^n} Q(s, z)|_{z=0} \tag{29}$$

The calculation of P_n it's possible if the correlation moments are obtained.

$$F_k(s) = \frac{\partial^k}{\partial z^k} Q(s, z)|_{z=1} \tag{30}$$

The equation obtained for quark and gluon jets is a Polya-Eggenberger distribution or negative binomial

$$P_q^m = \frac{k_p(k_p + 1...(k_p + m - 1))}{m!} (\frac{\bar{m}}{\bar{m} + k_p})^m (\frac{k_p}{\bar{m} + k_p})_p^k \tag{31}$$

Where k_p is determined by the ratio of probabilities of two elementary events, $\bar{m}$ is the average parton multiplicity

The Farry distribution is obtained for gluon jets[4]

$$P_m^g = \frac{k(k + 1...(m - 1))}{(m - k)!} \frac{1}{m!} (1 - \frac{1}{\bar{m}})^{m-1} \tag{32}$$

1.8 Hadronization

The successful description of MD in e^+ e^- annihilation processes suggests the idea of using this model to study proton interaction. It was found that the source of secondary hadrons is gluons, and the TSM was renamed The Gluon Dominance Model [15]. This model was implemented in two schemes, each with two stages. The first scheme takes into account gluon fission and the second does not.[6] The expressions for MD used to describe the first scheme are

$$P_n(s) = \sum_{k=1}^{MK} \frac{\bar{k}^k e^{-\bar{k}}}{k!} \sum_{m=k}^{MG} \frac{1}{\bar{m}^k} \frac{k(k+1)(k+2)...(m-1)}{(m-k)!} (1 - \frac{1}{\bar{m}})^{m-k} C_{\alpha m N}^{(n-2)} (\frac{\bar{n}_h}{N})^{(n-2)} (1 - \frac{\bar{n}^h}{N})^{\alpha m N (n-2)} \tag{33}$$

Here, k and m were the number of gluons at the initial state, the difference $n-2$ takes into account the two initial protons, α is the fraction of gluons that fragment into hadrons.
The equation for the second scheme is

$$P_n(s) = \sum_{m=1}^{ME} \frac{\bar{m}^m e^{-\bar{m}}}{m!} C_{mN}^{m-2} (\frac{\bar{n}^h}{N})^{n-2} (1 - \frac{\bar{n}^h}{N})^{mN-(n-2)} \tag{34}$$

1.9 Phi meson

The phi meson is composed of a strange quark and an anti-strange quark. Its strangeness is said to be hidden because it is equal to zero, since the S quark has a strangeness equal to -1 and the $\bar{S}$ equal to 1 [12].
The phi meson has been widely studied to understand the high densities reached in nuclei and neutron stars because it does not overlap with other resonances in the mass spectrum [10]. They are a source of study in relativistic heavy ion collisions since phi mesons are believed to interact weakly with hadronic matter, rapidly departing from the system, thereby accumulating information from the quark-gluon plasma. The properties of this hot, dense matter are of great interest to current science [12].
It was found that the number of phi mesons reconstructed from a dimuonic channel exceeds by a factor of 2 to 4 compared to that obtained from the K^+, K^- channel. It is speculated that the difference is due to the re-scattering of kaons in hadronic matter [2], a characteristic that muons almost do not possess because they practically do not interact with hadrons.
The dominant decay channel of the phi meson is (K^+K^-, K^0K^0), the decay to kaons is favored by the Okubo-Zweig-Iizuka (Ozi) rule, which states that Feynman diagrams in which the initial and final quark lines are disconnected are strongly suppressed in the strong interactions. [11]

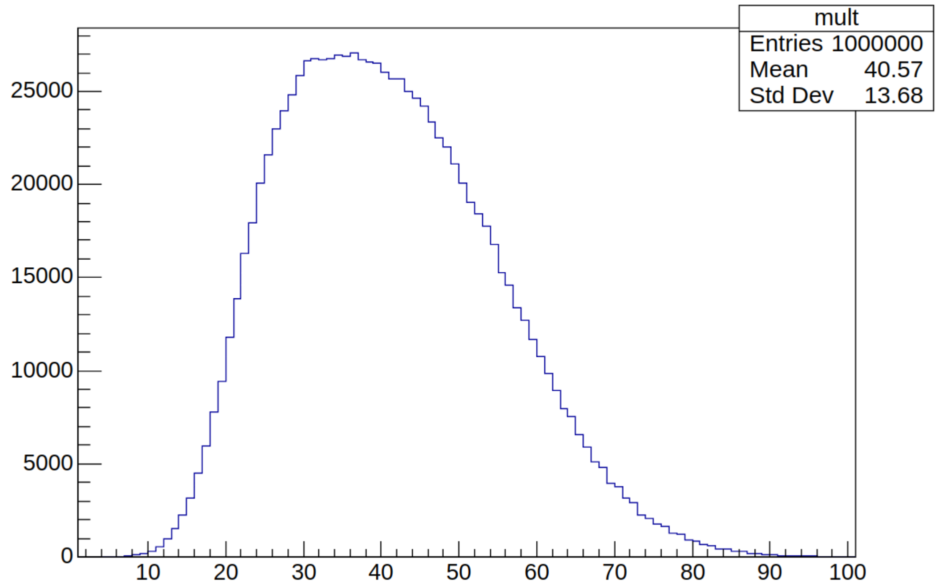

Figure 1: Distribution of multiplicity

2 Results

2.1 Multiplicity Distribution

A simulation was performed in Pythia considering a fixed-target proton-proton collision. Using the events and cuts reported in [7], the multiplicity of hadrons in the final state was extracted to analyze its behavior in the context of equation (34), excluding gluon fission effects. The resulting distributions are shown below
Additionally, the multiplicity distributions were studied for e^+e^- annihilation at various energies, applying a cut on Mg=20. The parameters obtained from these distributions at each energy are presented in the table below, together with the respective χ^2 values.

S(GeV)	μ	$\bar{m}$	$\bar{n}^h$	N	α	Ω	χ^2
14	20.0	0.083	4.465	27.738	0.967	1.997	2.79
22	10.0	0.85	4.859	28.0	0.383	1.998	1.72
34.8	10.0	2.0	5.213	28.0	0.303	1.997	9.16
43.6	20.0	2.0	5.6	28.0	0.348	1.987	22.87

2.2 Invariant Mass of Phi meson

Data obtained from a simulation in Pythia 8 and SPDroot, developed by the MLIT JINR team, were used. Only events with two kaons (K+ and K-) in the final state, candidates for originating from the phi meson decay, were selected. To calculate the invariant mass, taking the kaon and mass momentum. The invariant mass was calculated using the following equations

$$E = \sqrt{m^2 + \vec{p}^2} \tag{35}$$

$$(\vec{p_1} + \vec{p_2})^2 = S \tag{36}$$

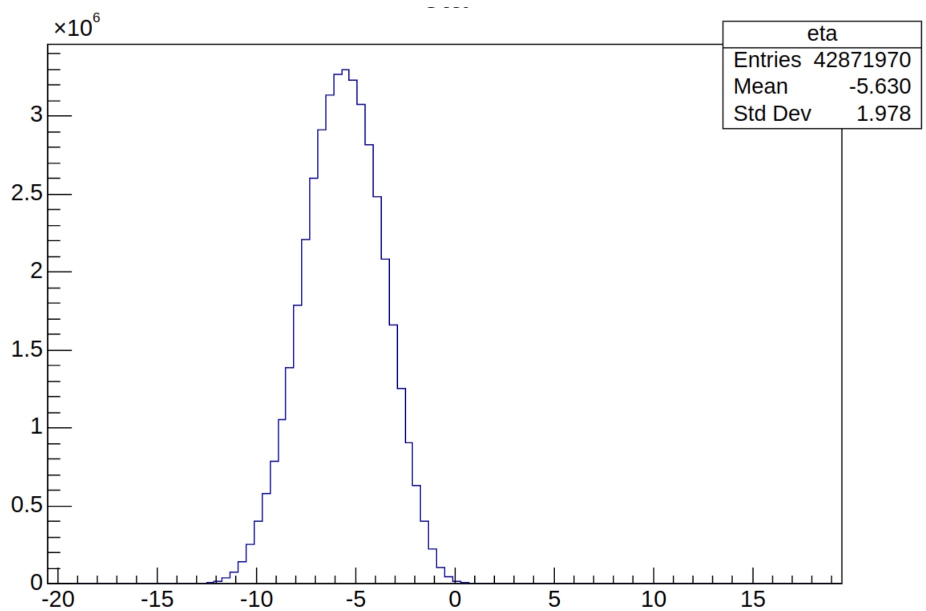

Figure 2: Distribution of pseudorapodity

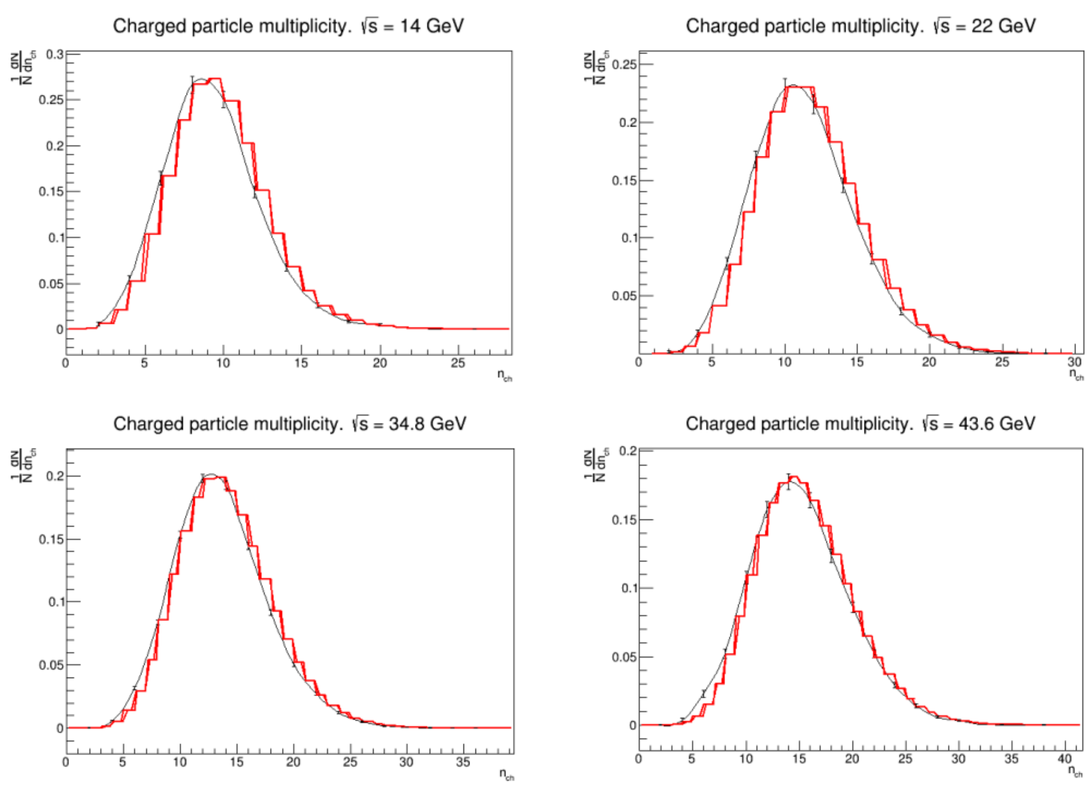

Figure 3: Multiplicity distributions of $e^{+}e^{-}$ annihilation at different energies

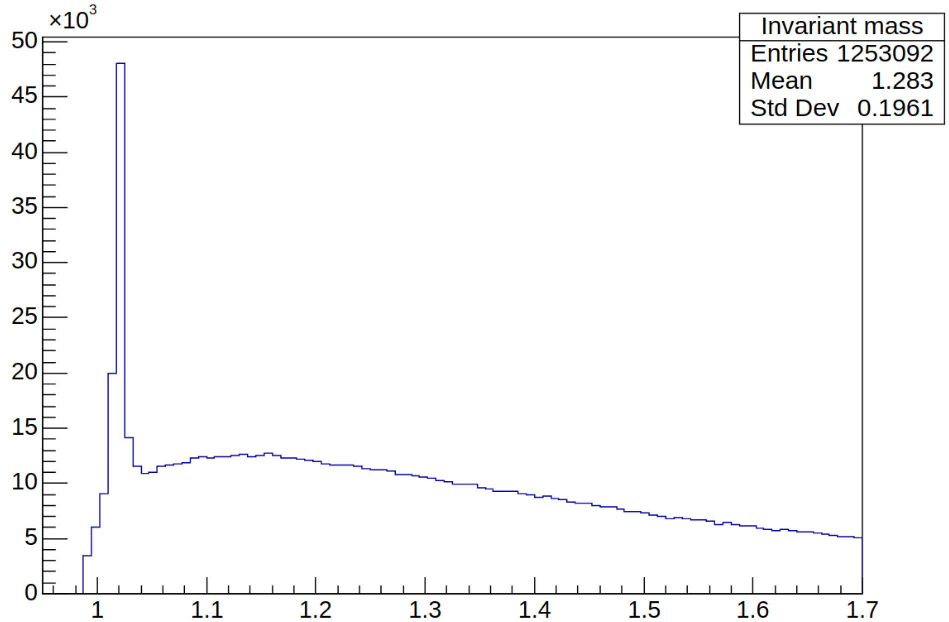

Figure 4: Invariant mass spectrum of K^+K^-

$$E = E_1 + E_2 = \sqrt{m^2 + \vec{p_1}^2} + \sqrt{m^2 + \vec{p_2}^2} \tag{37}$$

$$S = E^2 - (\vec{p_1}^2 + \vec{p_2}^2)^2 \tag{38}$$

$$S = 2m^2 + 2\sqrt{(p_1^2 + m^2)(p_2^2 + m^2)} - 2\vec{p_1}\vec{p_2} \tag{39}$$

The obtained invariant mass spectrum of the meson was separated into signal and background to separate the kaons that originated from a phi meson from those that originated from other processes. The signal was fitted using the relativistic Breit-Wigner equation(ref), and a degree 3 polynomial was used for the background. A total of 7 parameters were used.
Where k is a proportionality constant, M is the mass of the kaons, Γ is the width of the resonance related to its half-life according to $\Gamma = \frac{1}{\tau}$, E is the energy in the reference system of the center of mass that produces the resonance.
The parameters considered were k,M,Γ,P_1, P_2, P_3,P_4, the fit obtained was:
Histograms of zenith angle versus moment and Feynman x versus transverse moment were made.

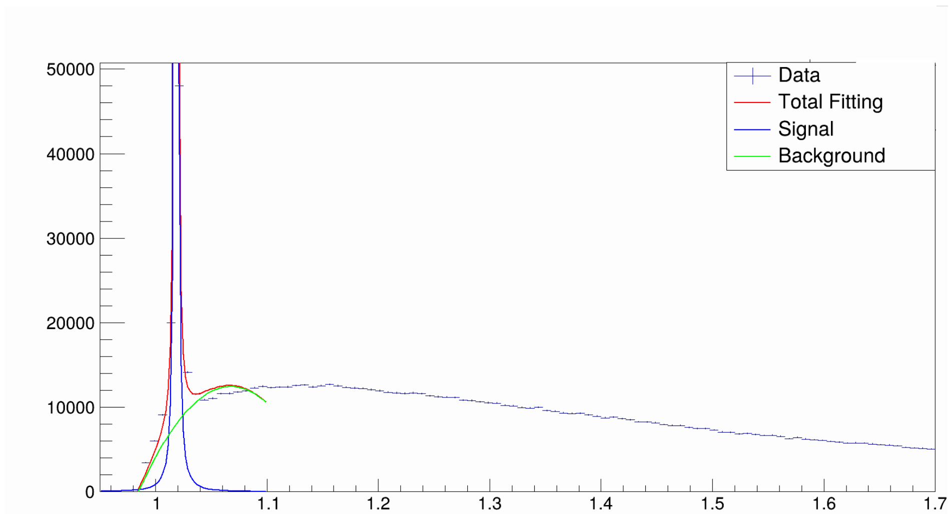

Figure 5: Invariant mass spectrum with signal and background distribution

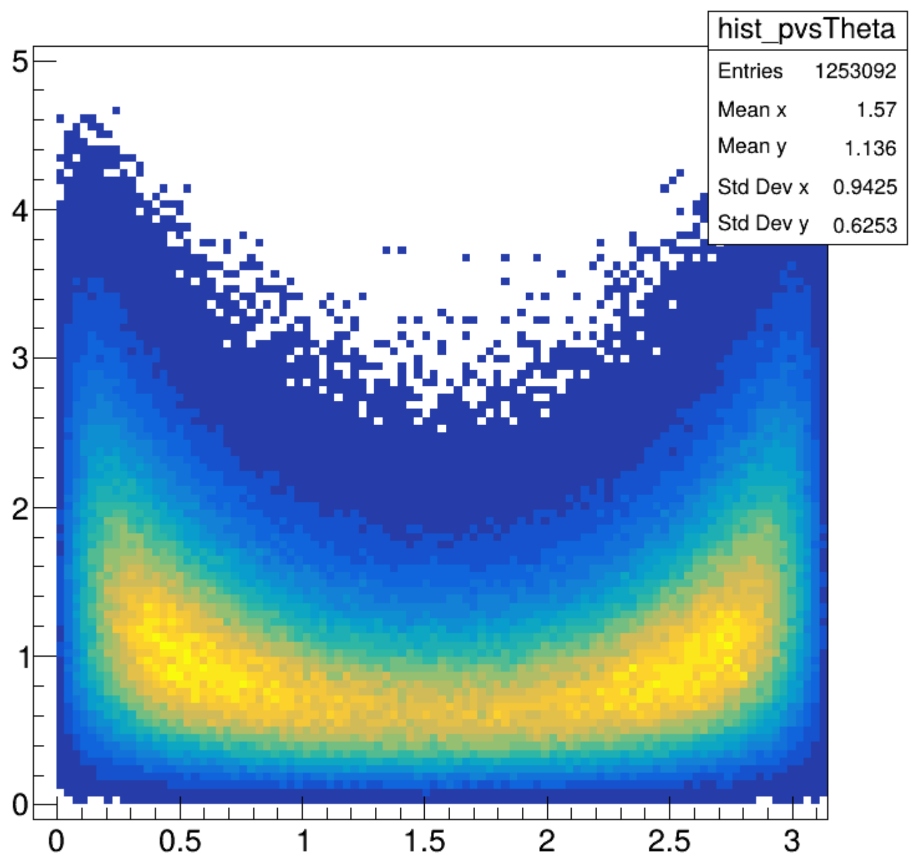

Figure 6: Distribution of total momentum vs zenithal angle

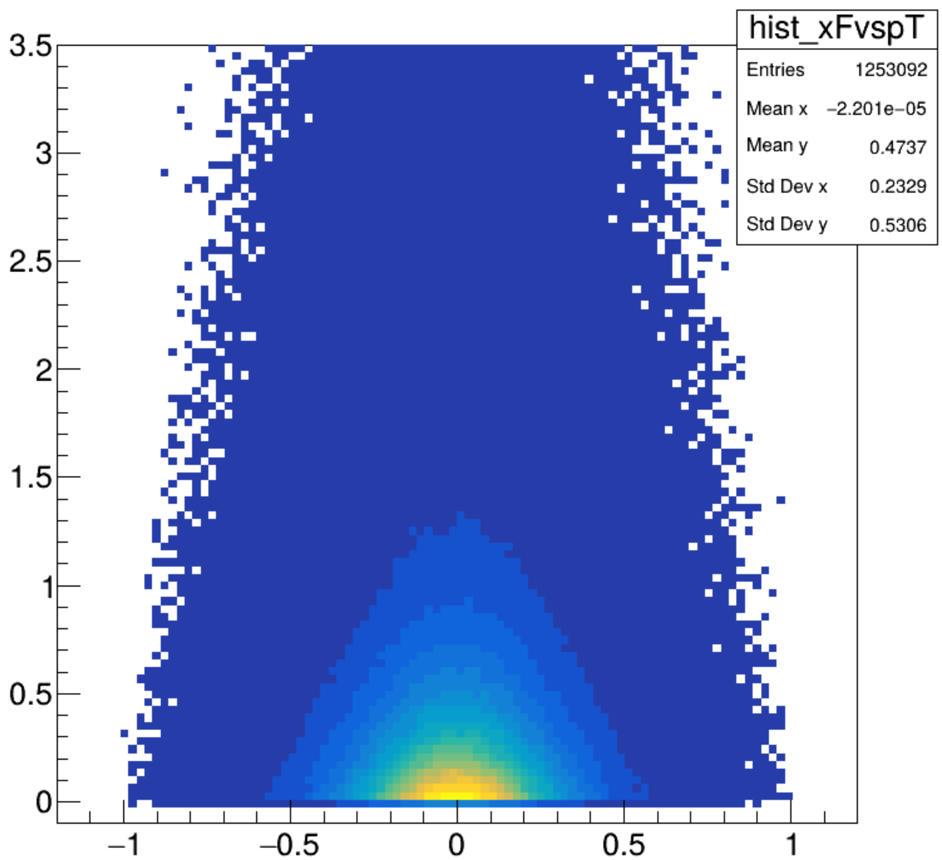

Figure 7: Distribution of x-Feynman vs transverse momentum

References

[1] Samuel Bright-Thonney and Benjamin Nachman. Investigating the topology dependence of quark and gluon jets. *Journal of High Energy Physics*, 2019(3):1–25, 2019.

[2] Daniel Cabrera Urbán and Manuel José Vicente Vacas. phi meson mass and decay width in nuclear matter. 2003.

[3] Jason Gallicchio and Matthew D Schwartz. Quark and gluon jet substructure. *Journal of High Energy Physics*, 2013(4):1–40, 2013.

[4] A Giovannini. Qcd jets as markov branching processes. *Nuclear Physics B*, 161(2-3):429–448, 1979.

[5] Nobuyuki Ikeda, Masao Nagasawa, and Shinzo Watanabe. Branching markov processes i. *Journal of Mathematics of Kyoto University*, 8(2):233–278, 1968.

[6] Elena Kokoulina and Andrey Kutov. Gluon dominance model. In *EPJ Web of Conferences*, volume 138, page 07007. EDP Sciences, 2017.

[7] ES Kokoulina, VA Nikitin, Yu P Petukhov, and A Ya Kutov. Proton interactions with high multiplicity. *Physics of Atomic Nuclei*, 75:664–667, 2012.

[8] Patrick T Komiske, Eric M Metodiev, and Jesse Thaler. An operational definition of quark and gluon jets. *Journal of High Energy Physics*, 2018(11):1–36, 2018.

[9] Kenichi Konishi, A Ukawa, and G Veneziano. A simple algorithm for qcd jets. *Physics Letters B*, 78(2-3):243–248, 1978.

[10] Arpita Mondal and Amruta Mishra. ϕ meson in nuclear matter and atomic nuclei. *arXiv preprint arXiv:2502.08320*, 2025.

[11] VP Nomokonov and MG Sapozhnikov. Experimental tests of the okubo-zweig-iizuka rule in hadron interactions. *arXiv preprint hep-ph/0204259*, 2002.

[12] Subrata Pal, CM Ko, and Zi-wei Lin. Phi meson production in relativistic heavy ion collisions. *Nuclear Physics A*, 707(3-4):525–539, 2002.

[13] John G Rushbrooke and Bryan R Webber. High energy antiparticle-particle reaction differences and annihilations. *Physics Reports*, 44(1):1–92, 1978.

[14] Tsutomu Sasao. Index generation functions: Tutorial. *Journal of Multiple-Valued Logic & Soft Computing*, 23, 2014.

[15] . ¡ img src="/get_item_image. asp? id= 17352056&img= fo_1_1. gif" align= absmiddle border= 0¿- 50 . , 75(3):343–343, 2012.