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FINAL REPORT ON THE START PROGRAMME

Application of the Glauber theory for elastic scattering of deuterons on deuterons

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Abstract

This paper is the final report of the START student program for the summer session 2023 and includes a consideration of the Glauber theory of multiple scattering, as well as its application to the calculation of elastic scattering amplitudes in the $pd \to pd$ and $dd \to dd$ processes. The objective of the work is to describe elastic dd-scattering according to Glauber with T-even P-even forces in the S-wave approximation. Analytical results are presented for the dd-scattering amplitude in terms of dN-amplitudes and in terms of nucleus-nucleus scattering. A comparison with experimental data on the differential scattering cross section at energies of 4.42 GeV/c is presented.

Introduction

SPD NICA at the first phase

The NICA ion collider project, developed at the Joint Institute for Nuclear Research, represents a step forward in the study of strong interactions and the properties of nuclear matter. Various types of collisions, including *pp*-, *dd*- and *pd*-collisions, provide the opportunity to study the spin dependence of strong interactions of energy regions in the center of mass NN system of the $\sqrt{s_{NN}} = 3 - 27$ GeV. This will make it possible to implement a geological program to study the structure of protons and deuterons and find the role of spin in strong interactions. The Spin Physics Detector (SPD) installation includes 4π-geometry, precision track and calorimetric detectors for particle identification [2], [1].

Of particular interest is communication in the average evening baryon numbers on the thresholds of the birth of double strangeness, charm and charm. This opens up new possibilities for studying multiquark flows and their connections with other structures.

Experiments with polarized beams of protons and deuterons are also of great interest, since they can provide insight into the development of the deuteron at short distances and its non-nucleon steps of freedom.

In addition, conducting experiments using the Spin Physics Detector (SPD) makes it possible to test the Standard Model and look for violations of time and parity invariance in single-spin scattering [1].

Thus, the NICA project represents an opportunity to conduct extensive nuclear experiments that will help defeat our ideas about the strong forces and the properties of nuclear matter.

Physics of the *dd*-channel

Spin observables of elastic *dd*-scattering can be used to check the spin-dependent amplitudes of elastic *pN*-scattering and to evaluate the influence of inelastic corrections on the *dd*-cross section at high energies $\sqrt{s_{NN}} = 53 - 63$ GeV. In addition, the spin-dependent amplitudes of elastic *pd* scattering can be used in the Glauber model to calculate *dd* scattering at low energies characteristic of the first phase of the NICA experiment, when the Gribov inelastic corrections are small. This approach will test the consistency of the model with experimental data and improve our understanding of nuclear structure and interactions.

At large scattering angles $\theta_{cm} \sim 90^\circ$ the processes $pd \to pd$ and $dd \to dd$ are sensitive to the short-range (six-quark) structure of the deuteron. Thus, measuring any observable of these processes at large values of θ_{cm} will be important for searching for non-nucleon degrees of freedom of the deuteron [1].

The collision processes of vectorially and tensorly polarized deuterons $\vec{d}\vec{d} \to dd$ are of great interest, since they contain information from the collisions $pn \to pn$ and $nn \to nn$ ($pp \to pp$) and shed light on the structure of NN interactions. Elastic *pp*- and *pn*-scattering at high energies $\sqrt{s} = 5 - 7$ GeV and large transferred momentum $|t| = 5 - 10$ GeV^2 is determined by the part of the NN interaction acting at a small distance between nucleons.

There are several features in the quantum chromodynamics of these processes.

The first feature is that the differential cross section at the angle $\theta_{cm} \sim 90^\circ$ obeys the quark counting rule of pertubative quantum chromodynamics, according to which $d\sigma_{pp}/dt \sim s^{-10}$, but in some region s diverges from the prediction in the form of oscillations. The second feature is the anomalous polarization asymmetry in hard pN-scattering at momentum $p = 11.75$ GeV/c – elastic scattering cross section with proton spins parallel and perpendicular to the scattering plane, approximately four times larger than the cross section with antiparallel spins. Thirdly, QCD predictions for hard NN scattering are associated with the phenomenon of color transparency, that is, a decrease in the absorption in the nuclear medium of hadrons (and mesons, baryons) produced in hard processes.

The combination of these three effects still raises questions. One explanation is the formation of octoquark resonances $uuds\bar{s}uud$ and $uudc\bar{c}uud$ in the s channel (which corresponds to the φ-meson and J/ψ charmonium) near the thresholds of the birth of strangeness and charm. This is supported by the agreement of the observed large variations in the spin correlations of elastic pp scattering with the formation of resonant states. Due to the fact that the elements of the transition matrix for J/ψ production in pn- and pp-collisions near the threshold have different spin-isospin structures, spin observables in elastic pn-scattering can provide valuable information . The task of obtaining these data in the energy range $\sqrt{s_{NN}} = 3 - 5$ GeV can be implemented on the NICA SPD [2], [1].

Problem

The first task of the work on the way to describing dd-scattering is to calculate the scattering amplitude in the case of T-even P-even forces in the s-wave approximation for the deutron wave function. For this purpose, Glauber's theory of multiple scattering in the forward hemisphere is applied: first in relation to the scattering of protons on deuterons $pd \to pd$, and then for dd-scattering.

System of units and notation

Below, the system of units is used everywhere, where $\hbar c = 0.1937\,\mathrm{mb} \cdot \mathrm{GeV}$

1 Basic principles of the Glauber theory of diffraction multiple scattering

An effective method for calculating the characteristics of hadronic processes at high energies and low momentum transfers is the Glauber – Sitenko diffraction theory of multiple scattering [3], [4]. This theory is based on the eikonal approximation, which is applicable at short wavelengths and small scattering angles. The eikonal approximation is well applicable to the amplitudes of elastic scattering of nucleons on nucleons (NN scattering), because they have a maximum when scattering at zero angle.

The scattering amplitude in the eikonal approximation can be written in the form (follows from the Lipmann – Schwinger equation):

$$f(\mathbf{q},k) = \frac{ik}{2\pi}\int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}}[1-e^{i\chi(\mathbf{b})}], \tag{1}$$

where the eikonal phase is related to the interaction potential $V(\mathbf{r})$

$$\chi(\mathbf{b}) = -\frac{\mu}{k}\int\limits_{-\infty}^{\infty} V(\mathbf{b}+z\hat{\mathbf{k}})\ \mathrm{d}z. \tag{2}$$

The assumption about the additivity of the potential energy of interaction of the external nucleon with the nucleons of the nucleus leads to the additivity of the eikonal phase:

$$\chi(\mathbf{b};\mathbf{s}_1,...,\mathbf{s}_A) = \sum_{i=1}^{A}\chi(\mathbf{b}-\mathbf{s}_i). \tag{3}$$

It is usually more convenient to use instead of the eikonal phase $\chi(\mathbf{b})$ the profile function $\gamma(\mathbf{b})$ corresponding to the scattering amplitude $f(\mathbf{q})$, which are related to each other friend using a two-dimensional Fourier transform:

$$f(\mathbf{q},k) = \frac{ik}{2\pi}\int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}}\gamma(\mathbf{b}), \tag{4}$$

$$\gamma(\mathbf{b}) = -\frac{i}{2\pi k}\int \mathrm{d}^2\mathbf{q}\, e^{-i\mathbf{qb}}f(\mathbf{b}) \tag{5}$$

When scattering on a compound particle (nucleus), the formula for calculating the scattering amplitude in Glauber's theory is naturally generalized:

$$F_{fi}(\mathbf{q}) = \frac{ip}{2\pi}\int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}}\int\prod_{j=1}^{A}\mathrm{d}^3\mathbf{r}_j\Psi_f^*(\mathbf{r}_1,...,\mathbf{r}_A)\left\{1-\prod_{l=1}^{A}[1-\gamma_l(\mathbf{b}-\mathbf{s}_l)]\right\}\Psi_i(\mathbf{r}_1,...,\mathbf{r}_A), \tag{6}$$

where $\Psi_i(\mathbf{r}_1,...,\mathbf{r}_A)$ and $\Psi_f(\mathbf{r}_1,...,\mathbf{r}_A)$ – are the initial and final wave functions, and the expression in curly brackets represents the scattering profile function on a composite particle, which is the probability of scattering on any of the particles in the nucleus. Formula (6) is the basic formula for calculating the scattering amplitude in Glauber's theory.

2 Formalism for elastic pd-scattering

2.1 Phenomenological parameterization of the NN-scattering amplitude

Elastic pN-scattering profile function with parameters (σ, α, a):

$$\gamma(\mathbf{b}) = \frac{\sigma(1 - i\alpha)}{4\pi a} \exp\left\{ -\frac{\mathbf{b}^2}{2a} \right\}. \tag{7}$$

Amplitude of elastic pN-scattering with parameters (σ, α, a):

$$f(\mathbf{q}, p) = \frac{ip}{2\pi} \int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}} \gamma(\mathbf{b}) = \frac{p\sigma}{4\pi}(i + \alpha) \exp\left\{ -\frac{a\mathbf{q}^2}{2} \right\}. \tag{8}$$

The pd-scattering profile function is expressed in terms of the probability that a proton interacts with at least one nucleon of a deuteron:

$$1 - (1 - \gamma_1)(1 - \gamma_2) = \gamma_1 + \gamma_2 - \gamma_1\gamma_2. \tag{9}$$

Let's substitute into the scattering amplitude:

$$F'_{fi}(\mathbf{q}, p) = \frac{ip}{2\pi} \int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}} \int \mathrm{d}^3\mathbf{r}_1\, \mathrm{d}^3\mathbf{r}_2 |\Psi(\mathbf{r}_1, \mathbf{r}_2)|^2 \times \\ \times \left(\gamma_1(\mathbf{b} - \mathbf{s}_1) + \gamma_2(\mathbf{b} - \mathbf{s}_2) - \gamma_1(\mathbf{b} - \mathbf{s}_1)\gamma_2(\mathbf{b} - \mathbf{s}_2) \right). \tag{10}$$

2.2 Spherically symmetric deuteron wave function (s-wave)

We will consider the s-wave as the deuteron wave function:

$$\Psi(\boldsymbol{\rho}) = Y_{00} U_0(\rho) = Y_{00} \sum_k A_k e^{-\varphi_k \rho^2}, \tag{11}$$

where $\boldsymbol{\rho} = \mathbf{r}_1 - \mathbf{r}_2$, $Y_{00} = 1/\sqrt{4\pi}$ – angular wave function, $U_0(\rho)$ – radial wave function of the deuteron.

The wave function is normalized:

$$\int |\Psi(\boldsymbol{\rho})|^2\, \mathrm{d}^2\boldsymbol{\rho} = \underbrace{\int \mathrm{d}\Omega\, Y_{00}^2}_{1} \int \mathrm{d}\rho\, \rho^2 U_0(\rho) = \sum_{k,l} \int_0^\infty A_k A_l e^{-(\varphi_k + \varphi_l)\rho^2} \rho^2\, \mathrm{d}\rho = \\ = \sum_{k,l} \frac{A_k A_l}{4} \sqrt{\frac{\pi}{(\varphi_k + \varphi_l)^3}} = 1. \tag{12}$$

For simplicity, let's move on to new coordinates in which

$$\begin{cases} \mathbf{r}' = \dfrac{\mathbf{r}_1 - \mathbf{r}_2}{2} = \dfrac{\boldsymbol{\rho}}{2}, \\ \mathbf{r}'_1 = \mathbf{r}' \\ \mathbf{r}'_2 = -\mathbf{r}'. \end{cases} \tag{13}$$

$$\Psi(\mathbf{r}') \equiv \sqrt{A^3}\Psi(\boldsymbol{\rho}) = \sqrt{A^3}Y_{00}\sum_k A_k e^{-4\varphi_k \mathbf{r}'^2}, \tag{14}$$

where the factor $\sqrt{A^3}$, which for the deuteron is equal to $\sqrt{2^3}$ (since $A = 2$), is introduced to preserve the normalization in new coordinates and is determined by the Jacobian of the transition:

$$\sqrt{2^3} = \sqrt{\frac{\partial(\boldsymbol{\rho})}{\partial(\mathbf{r}')}}, \quad \mathrm{d}^3\boldsymbol{\rho} = 2^3\,\mathrm{d}^3\mathbf{r}'. \tag{15}$$

The pd-scattering amplitude takes the form:

$$F_{pd}(\mathbf{q}) = \frac{ip}{2\pi}\int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}}\int \mathrm{d}^3\mathbf{r}'\,|\Psi(\mathbf{r}',-\mathbf{r}')|^2\Big[1-\big(1-\gamma_1(\mathbf{b}-\mathbf{s}')\big)\big(1-\gamma_2(\mathbf{b}+\mathbf{s}')\big)\Big] =$$

$$= \frac{ip}{2\pi}A^3Y_{00}^2\int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}}\sum_{k,l}A_kA_l\int_{-\infty}^{\infty}\mathrm{d}z'\int \mathrm{d}^2\mathbf{s}'\, e^{-(\varphi_k+\varphi_l)4(\mathbf{s}'^2+z'^2)}\big[\gamma_1+\gamma_2-\gamma_1\gamma_2\big]. \tag{16}$$

The terms with $(\gamma_1+\gamma_2)$ are responsible for single scattering, and $(-\gamma_1\gamma_2)$ for double scattering.

2.2.1 Single pd scattering

$$F_p^{(1)} = \frac{ip}{2\pi}A^3Y_{00}^2\int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}}\sum_{k,l}A_kA_l\int_{-\infty}^{\infty}\mathrm{d}z'\, e^{-(\varphi_k+\varphi_l)4z'^2}\times$$

$$\times\int \mathrm{d}^2\mathbf{s}'\, e^{-(\varphi_k+\varphi_l)4\mathbf{s}'^2}\left[\frac{\sigma_p(1-i\alpha_p)}{4\pi a_p}\right]e^{-\frac{(\mathbf{b}-\mathbf{s}')^2}{2a_p}} = A^3Y_{00}^2\sum_{k,l}A_kA_l(\sqrt{\pi}R_{kl})^3F_{kl}^{(1)}, \tag{17}$$

where

$$F_{kl}^{(1)} = \frac{ip}{2\pi}\left[\frac{\sigma_p(1-i\alpha_p)}{4\pi a_p}\right]\frac{1}{(\sqrt{\pi}R_{kl})^3}\int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}}\int_{-\infty}^{\infty}\mathrm{d}z'\, e^{-\frac{z'^2}{R_{kl}^2}}\int \mathrm{d}^2\mathbf{s}'\, e^{-\frac{\mathbf{s}'^2}{R_{kl}^2}}e^{-\frac{(\mathbf{b}-\mathbf{s}')^2}{2a_p}} \tag{18}$$

and $R_{kl} = \dfrac{1}{2\sqrt{\varphi_k+\varphi_l}}$.
Integrating (18) we obtain the amplitude:

$$F_{kl}^{(1)} = p\,\tilde{\alpha}_{kl}^{(1)}\, e^{-\beta_{kl}^{(1)}\mathbf{q}^2}, \tag{19}$$

where

$$\tilde{\alpha}_{kl}^{(1)} = -\frac{i}{2}\left[\frac{\sigma_p(i\alpha_p-1)}{2\pi}\right],$$

$$\beta_{kl}^{(1)} = \frac{2a_p+R_{kl}^2}{4} = \frac{1}{4}\left[2a_p+\frac{1}{4(\varphi_k+\varphi_l)}\right] = \frac{8a_p(\varphi_k+\varphi_l)+1}{16(\varphi_k+\varphi_l)}. \tag{20}$$

Thus, the amplitude of single scattering in the case of an s wave is expressed as follows:

$$F^{(1)} = p\sum_{k,l}\alpha_{kl}^{(1)}\, e^{-\beta_{kl}^{(1)}\mathbf{q}^2}, \tag{21}$$

where

$$\alpha_{kl}^{(1)} = -\frac{i}{2}\left[\frac{\sigma_p(i\alpha_p-1)}{2\pi}\right]A^3Y_{00}^2A_kA_l\left(\sqrt{\frac{\pi}{4(\varphi_k+\varphi_l)}}\right)^3,$$

$$\beta_{kl}^{(1)} = \frac{8a_p(\varphi_k+\varphi_l)+1}{16(\varphi_k+\varphi_l)}. \quad (22)$$

2.2.2 Double pd scattering

$$F^{(2)} = \frac{ip}{2\pi}A^3Y_{00}^2\int \mathrm{d}^2\mathbf{b}\,e^{i\mathbf{qb}}\sum_{k,l}A_kA_l\int\limits_{-\infty}^{\infty}\mathrm{d}z'\,e^{-4(\varphi_k+\varphi_l)z'^2}\times$$
$$\times\int\mathrm{d}^2\mathbf{s}'\,e^{-4(\varphi_k+\varphi_l)\mathbf{s}'^2}\left[\frac{\sigma_p(1-i\alpha_p)}{4\pi a_p}\right]\left[\frac{\sigma_n(1-i\alpha_n)}{4\pi a_n}\right]e^{-\frac{(\mathbf{b}-\mathbf{s}')^2}{2a_p}-\frac{(\mathbf{b}+\mathbf{s}')^2}{2a_n}} =$$
$$= A^3Y_{00}^2\sum_{k,l}A_kA_l(\sqrt{\pi}R_{kl})^3F_{kl}^{(1)}, \quad (23)$$

where

$$F_{kl}^{(2)} = \frac{ip}{2\pi}\left[\frac{\sigma_p(1-i\alpha_p)}{4\pi a_p}\right]\left[\frac{\sigma_n(1-i\alpha_n)}{4\pi a_n}\right]\times$$
$$\times\left[\frac{1}{(\sqrt{\pi}R_{kl})^3}\int\mathrm{d}^2\mathbf{b}\,e^{i\mathbf{qb}}\int\limits_{-\infty}^{\infty}\mathrm{d}z'\,e^{-\frac{z'^2}{R_{kl}^2}}\int\mathrm{d}^2\mathbf{s}'\,e^{-\frac{\mathbf{s}'^2}{R_{kl}^2}}e^{-\frac{(\mathbf{b}-\mathbf{s}')^2}{2a_p}-\frac{(\mathbf{b}+\mathbf{s}')^2}{2a_n}}\right], \quad (24)$$

and

$$R_{kl} = \frac{1}{2\sqrt{\varphi_k+\varphi_l}}.$$

Let's expand the subexponential expression:

$$-\frac{\mathbf{s}'^2}{R_{kl}^2}-\frac{(\mathbf{b}-\mathbf{s}')^2}{2a_p}-\frac{(\mathbf{b}+\mathbf{s}')^2}{2a_n} =$$
$$= -\mathbf{b}^2\left(\frac{1}{2a_p}+\frac{1}{2a_n}\right)-\mathbf{s}'^2\left(\frac{1}{2a_p}+\frac{1}{2a_n}+\frac{1}{R_{kl}^2}\right)+\mathbf{bs}'\left(\frac{1}{2a_p}-\frac{1}{2a_n}\right). \quad (25)$$

Integration over z' gives the factor $\sqrt{\pi}R_{kl}$, and integration over $\mathbf{s}'$ gives:

$$\left[\frac{1}{(\sqrt{\pi}R_{kl})^2}\int\mathrm{d}^2\mathbf{b}\,e^{i\mathbf{qb}}\int\mathrm{d}^2\mathbf{s}'\,e^{-\mathbf{b}^2\left(\frac{1}{2a_p}+\frac{1}{2a_n}\right)-\mathbf{s}'^2\left(\frac{1}{2a_p}+\frac{1}{2a_n}+\frac{1}{R_{kl}^2}\right)+\mathbf{bs}'\left(\frac{1}{2a_p}-\frac{1}{2a_n}\right)}\right] =$$

$$= \left[\frac{1}{(\sqrt{\pi}R_{kl})^2}\int\mathrm{d}^2\mathbf{b}\,e^{i\mathbf{qb}-\mathbf{b}^2\left(\frac{1}{2a_p}+\frac{1}{2a_n}\right)}\times\right.$$
$$\left.\times\,\pi\left[\frac{1}{2a_p}+\frac{1}{2a_n}+\frac{1}{R_{kl}^2}\right]^{-1}e^{\frac{\mathbf{b}^2}{4}\left(\frac{1}{2a_p}-\frac{1}{2a_n}\right)^2\left(\frac{1}{2a_p}+\frac{1}{2a_n}+\frac{1}{R_{kl}^2}\right)^{-1}}\right] = \circledcirc$$

Integration over $\mathbf{b}$:

$$\circledcirc = \left[\left[\frac{2a_pa_n}{2a_pa_n+(a_p+a_n)R_{kl}^2}\right]\int d^2\mathbf{b}\, e^{i\mathbf{qb}-\mathbf{b}^2\left(\frac{1}{2a_p}+\frac{1}{2a_n}\right)+\frac{\mathbf{b}^2}{4}\left(\frac{1}{2a_p}-\frac{1}{2a_n}\right)^2\left(\frac{1}{2a_p}+\frac{1}{2a_n}+\frac{1}{R_{kl}^2}\right)^{-1}}\right] \tag{26}$$

Let's simplify the factor before $\mathbf{b}^2$ in the exponent argument:

$$\begin{aligned}
&-\mathbf{b}^2\left(\frac{1}{2a_p}+\frac{1}{2a_n}\right)+\frac{\mathbf{b}^2}{4}\left(\frac{1}{2a_p}-\frac{1}{2a_n}\right)^2\left(\frac{1}{2a_p}+\frac{1}{2a_n}+\frac{1}{R_{kl}^2}\right)^{-1}=\\
&\qquad=-\mathbf{b}^2\left(\frac{a_p+a_n}{2a_pa_n}-\frac{1}{4}\frac{(2a_n-2a_p)^2}{(2a_p2a_n)^{\cancel{2}}}\frac{\cancel{2a_p2a_n}R_{kl}^2}{2a_p2a_n+2(a_p+a_n)R_{kl}^2}\right)=\\
&\qquad=-\mathbf{b}^2\left(\frac{1}{4a_pa_n}\frac{8a_pa_n(a_p+a_n)+4(a_p+a_n)^2R_{kl}^2-(a_n-a_p)^2R_{kl}^2}{2a_p2a_n+2(a_p+a_n)R_{kl}^2}\right)=\\
&\qquad=-\mathbf{b}^2\left(\frac{1}{4a_pa_n}\frac{8a_p^2a_n+8a_n^2a_p+(3a_p^2+10a_pa_n+3a_n^2)R_{kl}^2}{2a_p2a_n+2(a_p+a_n)R_{kl}^2}\right).
\end{aligned} \tag{27}$$

Finally, integrating over $\mathbf{b}$, we obtain

$$e^{-\frac{\mathbf{q}^2}{4}\left(\frac{(4a_pa_n)(2a_p2a_n+2(a_p+a_n)R_{kl}^2)}{8a_p^2a_n+8a_n^2a_p+(3a_p^2+10a_pa_n+3a_n^2)R_{kl}^2}\right)}\pi\left(\frac{(4a_pa_n)(2a_p2a_n+2(a_p+a_n)R_{kl}^2)}{8a_p^2a_n+8a_n^2a_p+(3a_p^2+10a_pa_n+3a_n^2)R_{kl}^2}\right). \tag{28}$$

In this way,

$$[\cdots]=16\pi a_p^2a_n^2\cdot\frac{e^{-\frac{\mathbf{q}^2}{4}\left(\frac{(4a_pa_n)(2a_p2a_n+2(a_p+a_n)R_{kl}^2)}{8a_p^2a_n+8a_n^2a_p+(3a_p^2+10a_pa_n+3a_n^2)R_{kl}^2}\right)}}{8a_p^2a_n+8a_n^2a_p+(3a_p^2+10a_pa_n+3a_n^2)R_{kl}^2}. \tag{29}$$

The amplitude of double pd scattering in the case of an s wave is equal to:

$$F^{(2)}=p\sum_{k,l}\alpha_{kl}^{(2)}\,e^{-\beta_{kl}^{(2)}\mathbf{q}^2}, \tag{30}$$

where

$$\begin{aligned}
\alpha_{kl}^{(2)}&=-\frac{i}{2}\left[\frac{\sigma_p(1-i\alpha_p)}{2\pi}\right]\left[\frac{\sigma_n(1-i\alpha_n)}{2\pi}\right]\frac{A^3}{2^2}Y_{00}^2\sum_{k,l}\frac{A_kA_l\cdot16\pi a_pa_n\cdot\left(\sqrt{\frac{\pi}{4(\varphi_k+\varphi_l)}}\right)^3}{8a_p^2a_n+8a_n^2a_p+(3a_p^2+10a_pa_n+3a_n^2)R_{kl}^2},\\
\beta_{kl}^{(2)}&=\frac{1}{4}\left(\frac{(4a_pa_n)(2a_p2a_n+2(a_p+a_n)R_{kl}^2)}{8a_p^2a_n+8a_n^2a_p+(3a_p^2+10a_pa_n+3a_n^2)R_{kl}^2}\right)
\end{aligned} \tag{31}$$

In the case $a_p=a_n=a$ we have:

$$[\cdots]=\left[\frac{\pi a^2}{a+R_{kl}^2}\right]e^{-\frac{\mathbf{q}^2a}{4}} \tag{32}$$

Similarly, for the amplitude of double scattering in the case of an s-wave under the condition $a_p = a_n = a$ we obtain:

$$F^{(2)} = p\sum_{k,l} \alpha_{kl}^{(2)}\, e^{-\beta_{kl}^{(2)}\mathbf{q}^2}, \tag{33}$$

where

$$\alpha_{kl}^{(2)} = -\frac{i}{2}\left[\frac{\sigma_p(i\alpha_p-1)}{2\pi}\right]\left[\frac{\sigma_n(i\alpha_n-1)}{2\pi}\right]\frac{A^3}{2^2}Y_{00}^2 A_k A_l\sqrt{\frac{\pi^3}{4(\varphi_k+\varphi_l)}}\cdot\left[\frac{1}{4a(\varphi_k+\varphi_l)+1}\right],$$

$$\beta_{kl}^{(2)} = \frac{a}{4}. \tag{34}$$

3 Elastic dd-scattering

3.1 Amplitude of dd-scattering in terms of Nd-scattering amplitudes

We look for the scattering amplitude $dd \to dd$ in the usual way:

$$F_{dd}(\mathbf{q}) = \frac{ip}{2\pi}\int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}}\int \mathrm{d}^3\mathbf{r}_1\,\mathrm{d}^3\mathbf{r}_2|\Psi(\mathbf{r}_1,\mathbf{r}_2)|^2\times$$
$$\times\left(\Gamma_1(\mathbf{b}-\mathbf{s}_1)+\Gamma_2(\mathbf{b}-\mathbf{s}_2)-\Gamma_1(\mathbf{b}-\mathbf{s}_1)\Gamma_2(\mathbf{b}-\mathbf{s}_2)\right). \tag{35}$$

Again we consider the s-wave (14). The scattering amplitude is calculated using a formula similar to (16):

$$F_{fi}(\mathbf{q}) = \frac{ip}{2\pi}A^3Y_{00}^2\int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}}\sum_{k,l}A_kA_l\int_{-\infty}^{\infty}\mathrm{d}z'\int \mathrm{d}^2\mathbf{s}'\, e^{-4(\varphi_k+\varphi_l)(\mathbf{s}'^2+z'^2)}\times$$
$$\times\left[\Gamma_1(\mathbf{b}-\mathbf{s}')+\Gamma_2(\mathbf{b}+\mathbf{s}')-\Gamma_1(\mathbf{b}-\mathbf{s}')\Gamma_2(\mathbf{b}+\mathbf{s}')\right]. \tag{36}$$

The incident particle this time is a deuteron, so as $\Gamma(\mathbf{b})$ we should consider the amplitude profile of Nd-scattering in the case of an s-wave (see (21), (33)):

$$F_{Nd}(\mathbf{q}) = F^{(1)} + F^{(2)} = p\sum_{k,l}\sum_{(\alpha_{kl},\beta_{kl})}\alpha_{kl}\, e^{-\beta_{kl}\mathbf{q}^2} \tag{37}$$

where

$$\alpha_{p;kl}^{(1)} = -\frac{i}{2}\left[\frac{\sigma_p(i\alpha_p-1)}{2\pi}\right]A^3Y_{00}^2A_kA_l\left(\sqrt{\frac{\pi}{4(\varphi_k+\varphi_l)}}\right)^3,$$

$$\alpha_{n;kl}^{(1)} = -\frac{i}{2}\left[\frac{\sigma_n(i\alpha_n-1)}{2\pi}\right]A^3Y_{00}^2A_kA_l\left(\sqrt{\frac{\pi}{4(\varphi_k+\varphi_l)}}\right)^3,$$

$$\alpha_{pn;kl}^{(2)} = -\frac{i}{2}\left[\frac{\sigma_p(i\alpha_p-1)}{2\pi}\right]\left[\frac{\sigma_n(i\alpha_n-1)}{2\pi}\right]\frac{A^3}{2^2}Y_{00}^2A_kA_l\sqrt{\frac{\pi^3}{4(\varphi_k+\varphi_l)}}\cdot\left[\frac{1}{4a(\varphi_k+\varphi_l)+1}\right],$$

$$\beta_{p;kl}^{(1)} = \beta_{n;kl}^{(1)} = \frac{8a(\varphi_k+\varphi_l)+1}{16(\varphi_k+\varphi_l)}.$$

$$\beta_{pn;kl}^{(2)} = \frac{a}{4}. \tag{38}$$

Nd-scattering amplitude profile in the case of an s-wave:

$$\Gamma(\mathbf{b}) = -\frac{i}{2\pi p}\int \mathrm{d}^2\mathbf{q}\left[p\sum_{k,l}\sum_{(\alpha_{kl},\beta_{kl})}\alpha_{kl}\,e^{-\beta_{kl}\mathbf{q}^2}\right]e^{-i\mathbf{q}\mathbf{b}} = -\frac{i}{2}\sum_{k,l}\sum_{(\alpha_{kl},\beta_{kl})}\frac{\alpha_{kl}}{\beta_{kl}}e^{-\frac{\mathbf{b}^2}{4\beta_{kl}}} \tag{39}$$

Contribution of $\Gamma_1+\Gamma_2$ to the amplitude of dd-scattering

$$F_p^{(1)} = \frac{ip}{2\pi}A^3Y_{00}^2\int \mathrm{d}^2\mathbf{b}\,e^{i\mathbf{q}\mathbf{b}}\sum_{i,j}A_iA_j\int_{-\infty}^{\infty}\mathrm{d}z'\,e^{-4(\varphi_i+\varphi_j)z'^2}\times$$
$$\times\int \mathrm{d}^2\mathbf{s}'\,e^{-4(\varphi_i+\varphi_j)\mathbf{s}'^2}\left[-\frac{i}{2}\sum_{k,l}\sum_{(\alpha_{kl},\beta_{kl})}\frac{\alpha_{kl}}{\beta_{kl}}e^{-\frac{(\mathbf{b}-\mathbf{s}')^2}{4\beta_{kl}}}\right] =$$
$$= A^3Y_{00}^2\sum_{i,j}A_iA_j(\sqrt{\pi}R_{ij})^3\sum_{k,l}\sum_{(\alpha_{kl},\beta_{kl})}F_{ij,kl}^{(1)}(\alpha_{kl},\beta_{kl}), \tag{40}$$

where

$$F_{ij,kl}^{(1)} = \frac{p}{4\pi}\frac{\alpha_{kl}}{\beta_{kl}}\left[\frac{1}{(\sqrt{\pi}R_{ij})^3}\int \mathrm{d}^2\mathbf{b}\,e^{i\mathbf{q}\mathbf{b}}\int_{-\infty}^{\infty}\mathrm{d}z'\,e^{-\frac{z'^2}{R_{ij}^2}}\int \mathrm{d}^2\mathbf{s}'\,e^{-\frac{\mathbf{s}'^2}{R_{ij}^2}-\frac{(\mathbf{b}-\mathbf{s}')^2}{4\beta_{kl}}}\right] =$$
$$= \frac{p}{\cancel{4\pi}}\frac{\alpha_{kl}}{\cancel{\beta_{kl}}}[\cancel{4\pi\beta_{kl}}]e^{-\mathbf{q}^2\frac{4\beta_{kl}+R_{ij}^2}{4}} = p\alpha_{kl}\,e^{-\mathbf{q}^2\frac{4\beta_{kl}+R_{ij}^2}{4}} = p\alpha_{kl}\,e^{-\mathbf{q}^2\frac{16\beta_{kl}(\varphi_j+\varphi_j)+1}{16(\varphi_j+\varphi_j)}}. \tag{41}$$

In total, the contribution of Γ_1 to the amplitude of dd-scattering in the case of an s-wave is equal to:

$$F_p^{(1)} = pA^3Y_{00}^2\sum_{i,j}A_iA_j\left(\sqrt{\frac{\pi}{4(\varphi_i+\varphi_j)}}\right)^3\sum_{k,l}\sum_{(\alpha_{kl},\beta_{kl})}\alpha_{kl}\,e^{-\beta_{kl}\mathbf{q}^2}e^{-\frac{1}{16(\varphi_j+\varphi_j)}\mathbf{q}^2}. \tag{42}$$

$$F_p^{(1)} = p\sum_{i,j}\xi_{ij}(\mathbf{q})\,e^{-\lambda_{ij}\mathbf{q}^2} \tag{43}$$

where

$$\xi_{ij}(\mathbf{q}) = A^3Y_{00}^2A_iA_j\left(\sqrt{\frac{\pi}{4(\varphi_i+\varphi_j)}}\right)^3\sum_{k,l}\sum_{(\alpha_{kl},\beta_{kl})}\alpha_{kl}\,e^{-\beta_{kl}\mathbf{q}^2}, \quad \lambda_{ij} = \frac{1}{16(\varphi_j+\varphi_j)}. \tag{44}$$

Contribution of $-\Gamma_1\Gamma_2$ to the amplitude of dd-scattering

$$F^{(2)} = \frac{ip}{2\pi}A^3Y_{00}^2\int \mathrm{d}^2\mathbf{b}\,e^{i\mathbf{q}\mathbf{b}}\sum_{i,j}A_iA_j\int_{-\infty}^{\infty}\mathrm{d}z'\,e^{-4(\varphi_i+\varphi_j)z'^2}\int \mathrm{d}^2\mathbf{s}'\,e^{-4(\varphi_i+\varphi_j)\mathbf{s}'^2}\times$$
$$\times\frac{1}{4}\left[\sum_{k_1,l_1}\sum_{(\alpha_{k_1l_1},\beta_{k_1l_1})}\frac{\alpha_{k_1l_1}}{\beta_{k_1l_1}}e^{-\frac{(\mathbf{b}-\mathbf{s}')^2}{4\beta_{k_1l_1}}}\right]\left[\sum_{k_2,l_2}\sum_{(\alpha_{k_2l_2},\beta_{k_2l_2})}\frac{\alpha_{k_2l_2}}{\beta_{k_2l_2}}e^{-\frac{(\mathbf{b}+\mathbf{s}')^2}{4\beta_{k_2l_2}}}\right] =$$
$$= A^3Y_{00}^2\sum_{i,j}A_iA_j\sum_{\substack{k_1,l_1\\k_2,l_2}}\sum_{\substack{(\alpha_{k_1l_1},\beta_{k_1l_1})\\(\alpha_{k_2l_2},\beta_{k_2l_2})}}F_{ij,k_1l_1,k_2l_2}^{(2)}(\alpha_{k_1l_1},\beta_{k_1l_1},\alpha_{k_2l_2},\beta_{k_2l_2}), \tag{45}$$

$$F^{(2)}_{ij,k_1l_1,k_2l_2} = \frac{ip}{8\pi}\frac{\alpha_{k_1l_1}}{\beta_{k_1l_1}}\frac{\alpha_{k_2l_2}}{\beta_{k_2l_2}}(\sqrt{\pi}R_{ij})^3 \times$$
$$\times \left[\left(\frac{1}{\sqrt{\pi}R_{ij}}\right)^3 \int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{qb}} \int\limits_{-\infty}^{\infty} \mathrm{d}z'\, e^{-\frac{z'^2}{R^2_{ij}}} \int \mathrm{d}^2\mathbf{s}'\, e^{-\frac{\mathbf{s}'^2}{R^2_{ij}}}\, e^{-\frac{(\mathbf{b}-\mathbf{s}')^2}{4\beta_{k_1l_1}} - \frac{(\mathbf{b}+\mathbf{s}')^2}{4\beta_{k_2l_2}}} \right]. \quad (46)$$

The integral in square brackets is calculated and equal (compare with (24) and (29), where $R_{kl} \to R_{ij}, a_p \to 2\beta_{k_1l_1}, a_n \to 2\beta_{k_2l_2}$):

$$[\,\cdots] = 16\pi\beta_{k_1l_1}\beta_{k_2l_2}\cdot 4(\varphi_i+\varphi_j)\cdot \frac{e^{-\frac{\mathbf{q}^2}{4}\left(\frac{\beta_{k_1l_1}+\beta_{k_2l_2}+16\beta_{k_1l_1}\beta_{k_2l_2}(\varphi_i+\varphi_j)}{1+4(\beta_{k_1l_1}+\beta_{k_2l_2})(\varphi_i+\varphi_j)}\right)}}{1+4(\beta_{k_1l_1}+\beta_{k_2l_2})(\varphi_i+\varphi_j)},$$

In total, the contribution of $-\Gamma_1\Gamma_2$ to the amplitude of dd-scattering in the case of an s-wave is equal to:

$$F^{(2)} = \frac{ip}{2}A^3Y^2_{00}\sum_{\substack{i,j\\k_1,l_1\\k_2,l_2}} A_iA_j \sum_{\substack{(\alpha_{k_1l_1},\beta_{k_1l_1})\\(\alpha_{k_2l_2},\beta_{k_2l_2})}} \alpha_{k_1l_1}\alpha_{k_2l_2}\sqrt{\frac{\pi^3}{4(\varphi_i+\varphi_j)}}\cdot \frac{e^{-\frac{\mathbf{q}^2}{4}\left(\frac{\beta_{k_1l_1}+\beta_{k_2l_2}+16\beta_{k_1l_1}\beta_{k_2l_2}(\varphi_i+\varphi_j)}{1+4(\beta_{k_1l_1}+\beta_{k_2l_2})(\varphi_i+\varphi_j)}\right)}}{1+4(\beta_{k_1l_1}+\beta_{k_2l_2})(\varphi_i+\varphi_j)}. \quad (47)$$

Amplitude of dd-scattering

The dd-scattering amplitude is equal according to (36), (42), (47)

$$F_{dd} = 2F^{(1)} + F^{(2)} =$$
$$= pA^3Y^2_{00}\sum_{i,j}A_iA_j\left(\sqrt{\frac{\pi}{4(\varphi_i+\varphi_j)}}\right)^3 \sum_{k,l}\sum_{(\alpha_{kl},\beta_{kl})}\alpha_{kl}\,e^{-\beta_{kl}\mathbf{q}^2}e^{-\frac{1}{16(\varphi_j+\varphi_j)}\mathbf{q}^2} +$$
$$+\frac{ip}{2}A^3Y^2_{00}\sum_{\substack{i,j\\k_1,l_1\\k_2,l_2}} A_iA_j \sum_{\substack{(\alpha_{k_1l_1},\beta_{k_1l_1})\\(\alpha_{k_2l_2},\beta_{k_2l_2})}} \alpha_{k_1l_1}\alpha_{k_2l_2}\sqrt{\frac{\pi^3}{4(\varphi_i+\varphi_j)}}\cdot \frac{e^{-\frac{\mathbf{q}^2}{4}\left(\frac{\beta_{k_1l_1}+\beta_{k_2l_2}+16\beta_{k_1l_1}\beta_{k_2l_2}(\varphi_i+\varphi_j)}{1+4(\beta_{k_1l_1}+\beta_{k_2l_2})(\varphi_i+\varphi_j)}\right)}}{1+4(\beta_{k_1l_1}+\beta_{k_2l_2})(\varphi_i+\varphi_j)}. \quad (48)$$

Let us take into account the relations (38) for the coefficients α_{kl} and β_{kl} and introduce the coefficients $\xi^0_{ij}, \lambda^0_{ij}$ using the formulas (66):

$$\alpha^{(1)}_{ij} = \left[\frac{\sigma_p(i+\alpha)}{8\pi}\right]\cdot 2\xi^0_{ij},$$
$$\alpha^{(2)}_{ij} = \left[\frac{\sigma_p(i+\alpha)}{8\pi}\right]^2\cdot 2i\xi^0_{ij}\left[\frac{\lambda^0_{ij}}{4}+a\right]^{-1},$$
$$\beta^{(1)}_{ij} = \frac{1}{4}\left[\frac{\lambda^0_{ij}}{4}+2a\right],$$
$$\beta^{(2)}_{ij} = \frac{a}{4}, \quad (49)$$

$$\xi^0_{ij} = \frac{\sqrt{\pi}}{4}\frac{C0_i C0_j}{(A0_i + A0_j)^{3/2}}, \quad \lambda^0_{ij} = \frac{1}{A0_i + A0_j}, \tag{50}$$

where the previous notations $A_i, A_j, \varphi_i, \varphi_j$ correspond to $C0_i, C0_j, A0_i, A0_j$.
Let's rewrite the expression (48)

$$
\begin{aligned}
&F_{dd} = p\sum_{ijkl}\xi^0_{ij}\xi^0_{kl}\Bigg[\\
&\left[\frac{\sigma(i+\alpha)}{8\pi}\right]\cdot 8\, e^{-\frac{a\mathbf{q}^2}{2}}\, e^{-\frac{\lambda^0_{ij}+\lambda^0_{kl}}{16}\mathbf{q}^2} +\\
&\left[\frac{\sigma(i+\alpha)}{8\pi}\right]^2\cdot 2i\, e^{-\frac{a\mathbf{q}^2}{4}}\left\{e^{-\frac{\lambda^0_{kl}\mathbf{q}^2}{16}}\left[\frac{\lambda^0_{ij}}{4}+a\right]^{-1} + e^{-\frac{\lambda^0_{ij}\mathbf{q}^2}{16}}\left[\frac{\lambda^0_{kl}}{4}+a\right]^{-1}\right\}\Bigg] +\\
&\quad +p\sum_{ijk_1l_1k_2l_2}\xi^0_{ij}\xi^0_{k_1l_1}\xi^0_{k_2l_2}\Bigg[\\
&\left[\frac{\sigma(i+\alpha)}{8\pi}\right]^2\cdot 8i\, e^{-\frac{\lambda^0_{ij}\mathbf{q}^2}{16}}\frac{1}{\frac{\lambda^0_{ij}}{4}+\frac{\lambda^0_{k_1l_1}+\lambda^0_{k_2l_2}}{16}+a}\exp\left\{-\frac{\mathbf{q}^2}{4}\frac{\left[\frac{\lambda^0_{k_1l_1}}{8}+a\right]\left[\frac{\lambda^0_{k_2l_2}}{8}+a\right]-\frac{\lambda^{0\,2}_{ij}}{16}}{\frac{\lambda^0_{ij}}{4}+\frac{\lambda^0_{k_1l_1}+\lambda^0_{k_2l_2}}{16}+a}\right\} +\\
&\left[\frac{\sigma(i+\alpha)}{8\pi}\right]^3\cdot 2\, e^{-\frac{a\mathbf{q}^2}{4}}\Bigg[\frac{3\left(\frac{\lambda^0_{ij}}{4}+a\right)+\frac{\lambda^0_{ij}+\lambda^0_{k_1l_1}}{4}}{\frac{\lambda^0_{k_2l_2}}{4}+a}\exp\left\{-\frac{\mathbf{q}^2}{4}\frac{-a\left[\frac{\lambda^0_{ij}}{4}+a\right]+\frac{\lambda^0_{ij}\lambda^0_{k_1l_1}}{16}}{3\left(\frac{\lambda^0_{ij}}{4}+a\right)+\frac{\lambda^0_{ij}+\lambda^0_{k_1l_1}}{4}}\right\} +\\
&\qquad + \frac{3\left(\frac{\lambda^0_{ij}}{4}+a\right)+\frac{\lambda^0_{ij}+\lambda^0_{k_2l_2}}{4}}{\frac{\lambda^0_{k_1l_1}}{4}+a}\exp\left\{-\frac{\mathbf{q}^2}{4}\frac{-a\left[\frac{\lambda^0_{ij}}{4}+a\right]+\frac{\lambda^0_{ij}\lambda^0_{k_2l_2}}{16}}{3\left(\frac{\lambda^0_{ij}}{4}+a\right)+\frac{\lambda^0_{ij}+\lambda^0_{k_2l_2}}{4}}\right\}\Bigg] +\\
&\left[\frac{\sigma(i+\alpha)}{8\pi}\right]^4\cdot(-4i)\, e^{-\frac{a\mathbf{q}^2}{8}}\left[\frac{\lambda^0_{ij}}{2}+a\right]^{-1}\left[\frac{\lambda^0_{k_1l_1}}{4}+a\right]^{-1}\left[\frac{\lambda^0_{k_2l_2}}{4}+a\right]^{-1}\Bigg]
\end{aligned}
\tag{51}
$$

3.2 Nucleus-nucleus scattering

In the paper of G. Alberi [11], an expression was obtained for the amplitude of *dd*-scattering, expressed in terms of the form factor and amplitude of nucleon-nucleon scattering:

$$F_{dd} = 8f(\mathbf{q})S^2\left(\tfrac{1}{2}\mathbf{q}\right) + \frac{2i}{\pi k}\left[4S\left(\tfrac{1}{2}\mathbf{q}\right)\int S(\mathbf{q}_1)f\left(\mathbf{q}_1+\tfrac{1}{2}\mathbf{q}\right)f\left(-\mathbf{q}_1+\tfrac{1}{2}\mathbf{q}\right)\mathrm{d}^2\mathbf{q}_1 + \right.$$
$$\left. + 2\int S^2(\mathbf{q}_1)f\left(\mathbf{q}_1+\tfrac{1}{2}\mathbf{q}\right)f\left(-\mathbf{q}_1+\tfrac{1}{2}\mathbf{q}\right)\mathrm{d}^2\mathbf{q}_1\right] -$$
$$-\frac{8}{\pi^2k^2}\int S(\mathbf{q}_1)S(\mathbf{q}_2)f\left(\tfrac{1}{2}\mathbf{q}-\mathbf{q}_1\right)f\left(\mathbf{q}_1+\mathbf{q}_2\right)f\left(\tfrac{1}{2}\mathbf{q}-\mathbf{q}_2\right)\mathrm{d}^2\mathbf{q}_1\,\mathrm{d}^2\mathbf{q}_2 -$$
$$-\frac{2i}{\pi^3k^3}\int S(\mathbf{q}_1)S(\mathbf{q}_2)f\left(\tfrac{1}{2}\mathbf{q}-\mathbf{q}_1-\mathbf{q}_3\right)f(\mathbf{q}_3)f(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3)f\left(\tfrac{1}{2}\mathbf{q}-\mathbf{q}_2-\mathbf{q}_3\right)\mathrm{d}^2\mathbf{q}_1\,\mathrm{d}^2\mathbf{q}_2\,\mathrm{d}^2\mathbf{q}_3. \tag{52}$$

Form factor:

$$S(\mathbf{q}) = S_0(\mathbf{q}) - S_2(\mathbf{q})[3(\mathbf{J}\cdot\hat{\mathbf{q}})^2 - 2], \tag{53}$$

Where

$$S_0(\mathbf{q}) = \int_0^\infty [u^2(r) + w^2(r)]j_0(qr)\,\mathrm{d}r, \tag{54}$$

$$S_2(\mathbf{q}) = \sqrt{2}\int_0^\infty w(r)\left[u(r) - \frac{w(r)}{\sqrt{8}}\right]j_2(qr)\,\mathrm{d}r, \tag{55}$$

where *s*- and *d*- wave functions are expanded into series:

$$u(r) = r\sum_{i=1}^{5} C0_i\, e^{-A0_i r^2}, \quad w(r) = r^3\sum_{k=1}^{5} C2_k\, e^{-A2_k r^2} \tag{56}$$

Form factor calculation

Let us calculate the amplitude, limiting ourselves to the contribution of S_0.

$$\int_0^\infty u^2(r)j_0(qr)\,\mathrm{d}r = \int_0^\infty r^2\sum_{i,j} C0_iC0_j\, e^{-(A0_i+A0_j)r^2}\cdot\frac{\sin qr}{qr}\,\mathrm{d}r =$$
$$= \sum_{i,j} C0_iC0_j\frac{1}{q}\int_0^\infty r\, e^{-(A0_i+A0_j)r^2}\underbrace{\sin qr}_{-\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}q}\cos qr}\,\mathrm{d}r =$$
$$= -\sum_{i,j} C0_iC0_j\frac{1}{q}\frac{\mathrm{d}}{\mathrm{d}q}\int_0^\infty e^{-(A0_i+A0_j)r^2}\cos qr\,\mathrm{d}r =$$

$$
\begin{aligned}
&= -\frac{1}{2}\sum_{i,j} C0_i C0_j \frac{1}{q}\frac{\mathrm{d}}{\mathrm{d}q}\int\limits_{-\infty}^{\infty} e^{-(A0_i+A0_j)r^2}\underbrace{\cos qr}_{\mathrm{Re}\,e^{iqr}}\,\mathrm{d}r = \\
&= -\frac{1}{2}\sum_{i,j} C0_i C0_j \frac{1}{q}\frac{\mathrm{d}}{\mathrm{d}q}\,\mathrm{Re}\int\limits_{-\infty}^{\infty} e^{-(A0_i+A0_j)r^2+iqr}\,\mathrm{d}r = \\
&= -\frac{1}{2}\sum_{i,j} C0_i C0_j \frac{1}{q}\frac{\mathrm{d}}{\mathrm{d}q}\sqrt{\frac{\pi}{(A0_i+A0_j)}}\, e^{-\frac{q^2}{4(A0_i+A0_j)}} = \\
&= \cancel{-}\frac{1}{\cancel{2}}\sum_{i,j} C0_i C0_j \frac{1}{\cancel{q}}\frac{\cancel{-2q}\sqrt{\pi}}{4(A0_i+A0_j)^{3/2}}\, e^{-\frac{q^2}{4(A0_i+A0_j)}}.
\end{aligned}
\tag{57}
$$

Thus,

$$
\int\limits_0^{\infty} u^2(r) j_0(qr)\,\mathrm{d}r = \frac{\sqrt{\pi}}{4}\sum_{i,j}\frac{C0_i C0_j}{(A0_i+A0_j)^{3/2}}\, e^{-\frac{q^2}{4(A0_i+A0_j)}}. \tag{58}
$$

$$
\begin{aligned}
\int\limits_0^{\infty} w^2(r) j_0(qr)\,\mathrm{d}r &= \int\limits_0^{\infty} r^6 \sum_{k,l} C2_k C2_l\, e^{-(A2_k+A2_k)r^2}\cdot\frac{\sin qr}{qr}\,\mathrm{d}r = \\
&= \int\limits_0^{\infty} r^6 \sum_{k,l} C2_k C2_l\, e^{-(A2_k+A2_k)r^2}\cdot\frac{\sin qr}{qr}\,\mathrm{d}r = \\
&= \sum_{k,l} C2_k C2_l \frac{1}{q}\int\limits_0^{\infty} r^5\, e^{-(A2_k+A2_k)r^2}\sin qr\,\mathrm{d}r = \\
&= -\frac{1}{2q}\frac{\mathrm{d}}{\mathrm{d}q}\sum_{k,l} C2_k C2_l \int\limits_{-\infty}^{\infty} r^4\, e^{-(A2_k+A2_k)r^2+iqr}\,\mathrm{d}r = \\
&= -\frac{1}{2q}\sum_{k,l} C2_k C2_l\,\frac{\mathrm{d}}{\mathrm{d}q} I(n=4, \alpha = A2_k + A2_l, \beta = iq) = \circledast,
\end{aligned}
\tag{59}
$$

where

$$
\begin{aligned}
I(n,\alpha,\beta) &= \int\limits_{-\infty}^{\infty} r^n\, e^{-\alpha r^2+\beta r}\,\mathrm{d}r = \int\limits_{-\infty}^{\infty} r^n\, e^{-\alpha\left(r-\frac{\beta}{2\alpha}\right)^2+\frac{\beta^2}{4\alpha}}\,\mathrm{d}r = e^{\frac{\beta^2}{4\alpha}}\int\limits_{-\infty}^{\infty}\left(x+\frac{\beta}{2\alpha}\right)^n e^{-\alpha x^2}\,\mathrm{d}x = \\
&= e^{\frac{\beta^2}{4\alpha}}\sum_{k=0}^{n}\binom{n}{k}\left(\frac{\beta}{2\alpha}\right)^{n-k}\int\limits_{-\infty}^{\infty} x^k\, e^{-\alpha x^2}\,\mathrm{d}x = e^{\frac{\beta^2}{4\alpha}}\Big[\sum_{k=0,2\dots}^{n} + \sum_{k=1,3\dots}^{n}\Big]\binom{n}{k}\left(\frac{\beta}{2\alpha}\right)^{n-k}\int\limits_{-\infty}^{\infty} x^k\, e^{-\alpha x^2}\,\mathrm{d}x.
\end{aligned}
\tag{60}
$$

If $k = 2m+1$ is odd, then the integral with symmetric limits will contain the product of an odd function and an even exponential, so the integral will be equal to zero. Therefore, we are only interested in the case of even exponent $k = 2m$:

$$\int_{-\infty}^{\infty} x^{2m}\, e^{-\alpha x^2}\, \mathrm{d}x = 2\int_0^{\infty} x^{2m}\, e^{-\alpha x^2}\, \mathrm{d}x = \left|\xi = \alpha x^2, \mathrm{d}\xi = 2\alpha x\, \mathrm{d}x = 2\sqrt{\alpha\xi}\, \mathrm{d}x\right| =$$

$$= \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{\alpha}} \int_0^{\infty} \frac{1}{\sqrt{\xi}} \frac{\xi^m}{\alpha^m}\, e^{-\xi}\, \mathrm{d}\xi = \frac{1}{\sqrt{\alpha}} \int_0^{\infty} \xi^{m+\frac{1}{2}-1}\, e^{-\xi}\, \mathrm{d}\xi = \frac{1}{\sqrt{\alpha}} \frac{1}{\alpha^m}\, \Gamma\left(m + \frac{1}{2}\right) =$$

$$= \frac{(2m-1)!!}{2^m \alpha^m} \sqrt{\frac{\pi}{\alpha}}. \quad (61)$$

The integral $I(n, \alpha, \beta)$ is equal to

$$I(n, \alpha, \beta) = e^{\frac{\beta^2}{4\alpha}} \sum_{m=0}^{n/2} \binom{n}{2m} \left(\frac{\beta}{2\alpha}\right)^{n-2m} \frac{(2m-1)!!}{2^m \alpha^m} \sqrt{\frac{\pi}{\alpha}}. \quad (62)$$

$$\circledast = -\frac{1}{2q} \sum_{k,l} C2_k C2_l\, \frac{\mathrm{d}}{\mathrm{d}q} I(n = 4, \alpha = A2_k + A2_l, \beta = iq) =$$

$$= -\frac{1}{2q} \sum_{k,l} C2_k C2_l\, \frac{\mathrm{d}}{\mathrm{d}q}\, e^{\frac{(iq)^2}{4(A2_k + A2_l)}} \sum_{m=0}^{2} \binom{4}{2m} \left(\frac{iq}{2(A2_k + A2_l)}\right)^{4-2m} \frac{(2m-1)!!}{2^m \alpha^m} \sqrt{\frac{\pi}{(A2_k + A2_l)}} =$$

$$= -\frac{\sqrt{\pi}}{32} \sum_{k,l} \frac{C2_k C2_l}{(A2_k + A2_l)^{5/2}} \frac{1}{q} \frac{\mathrm{d}}{\mathrm{d}q} \left\{ e^{-\frac{q^2}{4(A2_k + A2_l)}} \left[\frac{q^4}{(A2_k + A2_l)^2} - \frac{12q^2}{A2_k + A2_l} + 12 \right] \right\} =$$

$$= -\frac{\sqrt{\pi}}{16} \sum_{k,l} \frac{C2_k C2_l}{(A2_k + A2_l)^{5/2}} \frac{\mathrm{d}}{\mathrm{d}(q^2)} \left\{ e^{-\frac{q^2}{4(A2_k + A2_l)}} \left[\frac{q^4}{(A2_k + A2_l)^2} - \frac{12q^2}{A2_k + A2_l} + 12 \right] \right\} =$$

$$= \frac{\sqrt{\pi}}{64} \sum_{k,l} \frac{C2_k C2_l}{(A2_k + A2_l)^{11/2}} \left[q^4 - 20q^2(A2_k + A2_l) + 60(A2_k + A2_l)^2\right] e^{-\frac{q^2}{4(A2_k + A2_l)}}. \quad (63)$$

$$\int_0^{\infty} w^2(r) j_0(qr)\, \mathrm{d}r =$$

$$= \frac{\sqrt{\pi}}{64} \sum_{k,l} \frac{C2_k C2_l}{(A2_k + A2_l)^{11/2}} \left[q^4 - 20q^2(A2_k + A2_l) + 60(A2_k + A2_l)^2\right] e^{-\frac{q^2}{4(A2_k + A2_l)}}. \quad (64)$$

The total form factor is

$$S_0(\mathbf{q}) = \int_0^{\infty} [u^2(r) + w^2(r)] j_0(qr)\, \mathrm{d}r = \frac{\sqrt{\pi}}{4} \sum_{i,j} \left[\frac{C0_i C0_j}{(A0_i + A0_j)^{3/2}}\, e^{-\frac{q^2}{4(A0_i + A0_j)}} + \right.$$

$$\left. + \frac{1}{16} \frac{C2_i C2_j}{(A2_i + A2_j)^{11/2}} \left[q^4 - 20q^2(A2_i + A2_j) + 60(A2_i + A2_j)^2\right] e^{-\frac{q^2}{4(A2_i + A2_j)}} \right] =$$

$$= \sum_{i,j} \left[\xi^0_{ij}\, e^{-\frac{\lambda^0_{ij} q^2}{4}} + \xi^2_{ij} \left[(\lambda^2_{ij} q^2)^2 - 20\lambda^2_{ij} q^2 + 60\right] e^{-\frac{\lambda^2_{ij} q^2}{4}} \right], \quad (65)$$

where

$$\xi^{\#}_{ij} = \frac{\sqrt{\pi}}{4\cdot 4^{\#}} \frac{C\#_i C\#_j}{(A\#_i + A\#_j)^{3/2+\#}}, \quad \lambda^{\#}_{ij} = \frac{1}{A\#_i + A\#_j}. \tag{66}$$

Contribution to the s-wave form factor

$$S_u(\mathbf{q}) = \int_0^\infty u^2(r) j_0(qr)\,\mathrm{d}r = \sum_{i,j} \xi^0_{ij}\, e^{-\frac{\lambda^0_{ij} q^2}{4}}, \tag{67}$$

Contribution to the d-wave form factor

$$S_w(\mathbf{q}) = \int_0^\infty w^2(r) j_0(qr)\,\mathrm{d}r = \sum_{i,j} \xi^2_{ij} \left[(\lambda^2_{ij} q^2)^2 - 20\lambda^2_{ij} q^2 + 60\right] e^{-\frac{\lambda^2_{ij} q^2}{4}} =$$
$$= \sum_{i,j} \xi^2_{ij} \left[(\lambda^2_{ij} q^2)^2 - 20\lambda^2_{ij} q^2\right] e^{-\frac{\lambda^2_{ij} q^2}{4}} + 60 \sum_{i,j} \xi^2_{ij}\, e^{-\frac{\lambda^2_{ij} q^2}{4}}, \tag{68}$$

Let's return to the formula (52)

$$F_{dd} = 8 f(\mathbf{q}) S^2\left(\tfrac{1}{2}\mathbf{q}\right) + \frac{2i}{\pi k}\Bigg[4S\left(\tfrac{1}{2}\mathbf{q}\right) \int S(\mathbf{q}_1) f\left(\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) f\left(-\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) \mathrm{d}^2\mathbf{q}_1 +$$
$$+ 2\int S^2(\mathbf{q}_1) f\left(\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) f\left(-\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) \mathrm{d}^2\mathbf{q}_1\Bigg] -$$
$$- \frac{8}{\pi^2 k^2} \int S(\mathbf{q}_1) S(\mathbf{q}_2) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_1\right) f\left(\mathbf{q}_1 + \mathbf{q}_2\right) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_2\right) \mathrm{d}^2\mathbf{q}_1\,\mathrm{d}^2\mathbf{q}_2 -$$
$$- \frac{2i}{\pi^3 k^3} \int S(\mathbf{q}_1) S(\mathbf{q}_2) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_1 - \mathbf{q}_3\right) f(\mathbf{q}_3) f(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_2 - \mathbf{q}_3\right) \mathrm{d}^2\mathbf{q}_1\,\mathrm{d}^2\mathbf{q}_2\,\mathrm{d}^2\mathbf{q}_3. \tag{69}$$

$$f(\mathbf{q}) S^2\left(\tfrac{1}{2}\mathbf{q}\right) = k\left[\frac{\sigma(i+\alpha)}{8\pi}\right] e^{-\frac{a\mathbf{q}^2}{2}} \sum_{ijkl} \xi^0_{ij} \xi^0_{kl}\, e^{-\frac{\lambda^0_{ij} + \lambda^0_{kl}}{16}\mathbf{q}^2},$$

$$\int S(\mathbf{q}_1) f\left(\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) f\left(-\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) \mathrm{d}^2\mathbf{q}_1 = k^2\left[\frac{\sigma(i+\alpha)}{8\pi}\right]^2 e^{-\frac{a\mathbf{q}^2}{4}} \pi \sum_{ij} \xi^0_{ij} \left[\frac{\lambda^0_{ij}}{4} + a\right]^{-1},$$

$$\int S^2(\mathbf{q}_1) f\left(\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) f\left(-\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) \mathrm{d}^2\mathbf{q}_1 = k^2\left[\frac{\sigma(i+\alpha)}{8\pi}\right]^2 e^{-\frac{a\mathbf{q}^2}{4}} \pi \sum_{ijkl} \xi^0_{ij} \xi^0_{kl} \left[\frac{\lambda^0_{ij} + \lambda^0_{kl}}{4} + a\right]^{-1},$$

$$\int S(\mathbf{q}_1) S(\mathbf{q}_2) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_1\right) f\left(\mathbf{q}_1 + \mathbf{q}_2\right) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_2\right) \mathrm{d}^2\mathbf{q}_1\,\mathrm{d}^2\mathbf{q}_2 =$$
$$= k^3\left[\frac{\sigma(i+\alpha)}{8\pi}\right]^3 e^{-\frac{a\mathbf{q}^2}{4}} \pi^2 \sum_{ijkl} \frac{\xi^0_{ij} \xi^0_{kl}}{\left[\frac{\lambda^0_{ij}}{4} + a\right]\left[\frac{\lambda^0_{kl}}{4} + a\right] - \frac{a^2}{4}} \exp\left\{\frac{\frac{a^2\mathbf{q}^2}{16}\left(\frac{\lambda^0_{ij} + \lambda^0_{kl}}{4} + a\right)}{\left[\frac{\lambda^0_{ij}}{4} + a\right]\left[\frac{\lambda^0_{kl}}{4} + a\right] - \frac{a^2}{4}}\right\},$$

$$\int S(\mathbf{q}_1) S(\mathbf{q}_2) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_1 - \mathbf{q}_3\right) f(\mathbf{q}_3) f(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_2 - \mathbf{q}_3\right) \mathrm{d}^2\mathbf{q}_1\,\mathrm{d}^2\mathbf{q}_2\,\mathrm{d}^2\mathbf{q}_3 =$$
$$= k^4\left[\frac{\sigma(i+\alpha)}{8\pi}\right]^4 e^{-\frac{a\mathbf{q}^2}{8}} \pi^3 \sum_{ijkl} \xi^0_{ij} \xi^0_{kl} \frac{1}{2a}\left[\frac{\lambda^0_{ij}}{4} + \frac{a}{2}\right]^{-1}\left[\frac{\lambda^0_{kl}}{4} + \frac{a}{2}\right]^{-1}. \tag{70}$$

Substituting (52) into the formula, we obtain in the case of the s-wave approximation the following expression for the amplitude F_{dd}:

$$
\begin{aligned}
&F_{dd} = k \sum_{ijkl} \xi^0_{ij} \xi^0_{kl} \Bigg[\\
&\left[\frac{\sigma(i+\alpha)}{8\pi}\right] \cdot 8\, e^{-\frac{a\mathbf{q}^2}{2}} e^{-\frac{\lambda^0_{ij}+\lambda^0_{kl}}{16}\mathbf{q}^2} + \\
&\left[\frac{\sigma(i+\alpha)}{8\pi}\right]^2 \cdot 4i\, e^{-\frac{a\mathbf{q}^2}{4}} \left\{ e^{-\frac{\lambda^0_{kl}\mathbf{q}^2}{16}} \left[\frac{\lambda^0_{ij}}{4} + a\right]^{-1} + e^{-\frac{\lambda^0_{ij}\mathbf{q}^2}{16}} \left[\frac{\lambda^0_{kl}}{4} + a\right]^{-1} + \left[\frac{\lambda^0_{ij}+\lambda^0_{kl}}{4} + a\right]^{-1} \right\} + \\
&\left[\frac{\sigma(i+\alpha)}{8\pi}\right]^3 \cdot (-8)\, e^{-\frac{a\mathbf{q}^2}{4}} \frac{1}{\left[\frac{\lambda^0_{ij}}{4} + a\right]\left[\frac{\lambda^0_{kl}}{4} + a\right] - \frac{a^2}{4}} \exp\left\{ \frac{\frac{a^2\mathbf{q}^2}{16}\left(\frac{\lambda^0_{ij}+\lambda^0_{kl}}{4} + a\right)}{\left[\frac{\lambda^0_{ij}}{4} + a\right]\left[\frac{\lambda^0_{kl}}{4} + a\right] - \frac{a^2}{4}} \right\} + \\
&\left[\frac{\sigma(i+\alpha)}{8\pi}\right]^4 \cdot (-i)\, e^{-\frac{a\mathbf{q}^2}{8}} \frac{1}{a} \left[\frac{\lambda^0_{ij}}{4} + \frac{a}{2}\right]^{-1} \left[\frac{\lambda^0_{kl}}{4} + \frac{a}{2}\right]^{-1} \Bigg].
\end{aligned}
\tag{71}
$$

4 Results

Experiment & theory

To compare with experimental data, we need to move from amplitude to differential cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{p^2}{\pi}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{p^2}{\pi}|F_{dd}|^2 \tag{72}$$

When calculating the differential scattering cross section, we use the CD-Bonn model to describe the s-wave function of the deuteron with the parameters presented in the Appendix (see also [9]). As parameters σ, α, a of the NN amplitudes (8) we use the values from the papers of Bassel R., Willkin C. [8] and Alberi G. at al. [11]. The work compared theoretical expressions for differential interaction cross sections, specified by amplitudes using the formulas (51) and (71), with experimental data from the paper of Devensky [12].

Table 1: Differential cross sections of elastic d-d scattering at 4.6 GeV/c

$\lvert t\rvert, \left(\frac{\text{GeV}}{c}\right)^2$	$\frac{d\sigma}{dt}, \text{mb}\Big/\left(\frac{\text{GeV}}{c}\right)^2$
0.0275	232 ± 15
0.0287	216 ± 16
0.03	205 ± 14
0.032	167 ± 10
0.0353	136 ± 7
0.0381	107 ± 7
0.0411	84.4 ± 5.2
0.0456	69.3 ± 4.6
0.0485	67.6 ± 6.3
0.0518	54.4 ± 3.4
0.0539	44 ± 2.4
0.0552	40.6 ± 2.1
0.0605	29.3 ± 1.6
0.0646	24.6 ± 1.8
0.0699	19.8 ± 1.1
0.0764	14.7 ± 1.3
0.0821	10.3 ± 0.7
0.0952	5.1 ± 0.4
0.105	2.7 ± 0.9
0.114	2.5 ± 0.5
0.119	1.5 ± 0.4
0.123	1.6 ± 0.5
0.126	1.3 ± 0.2
0.13	0.78 ± 0.13
0.146	0.32 ± 0.1

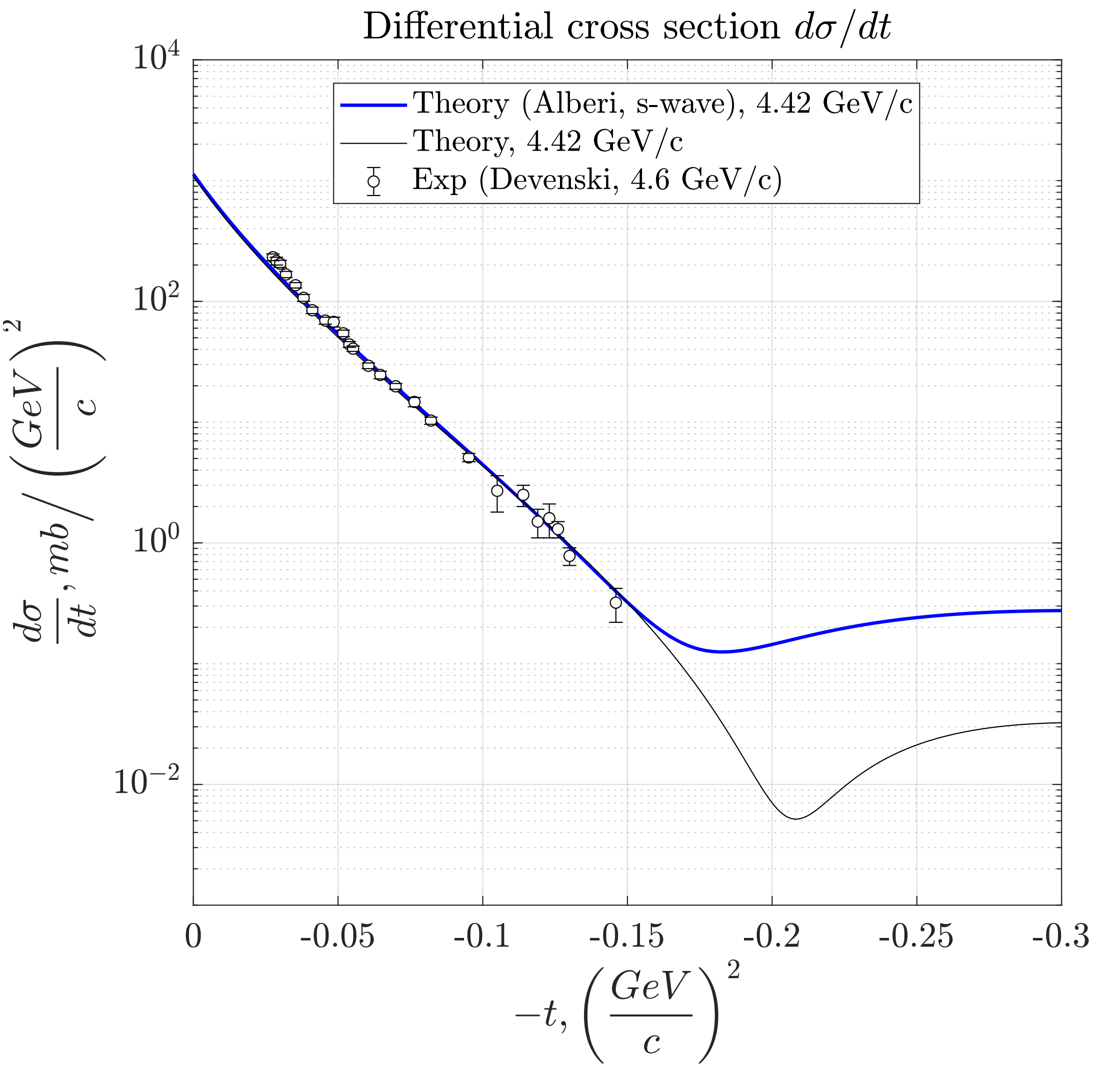

Figure 1: Differential cross section

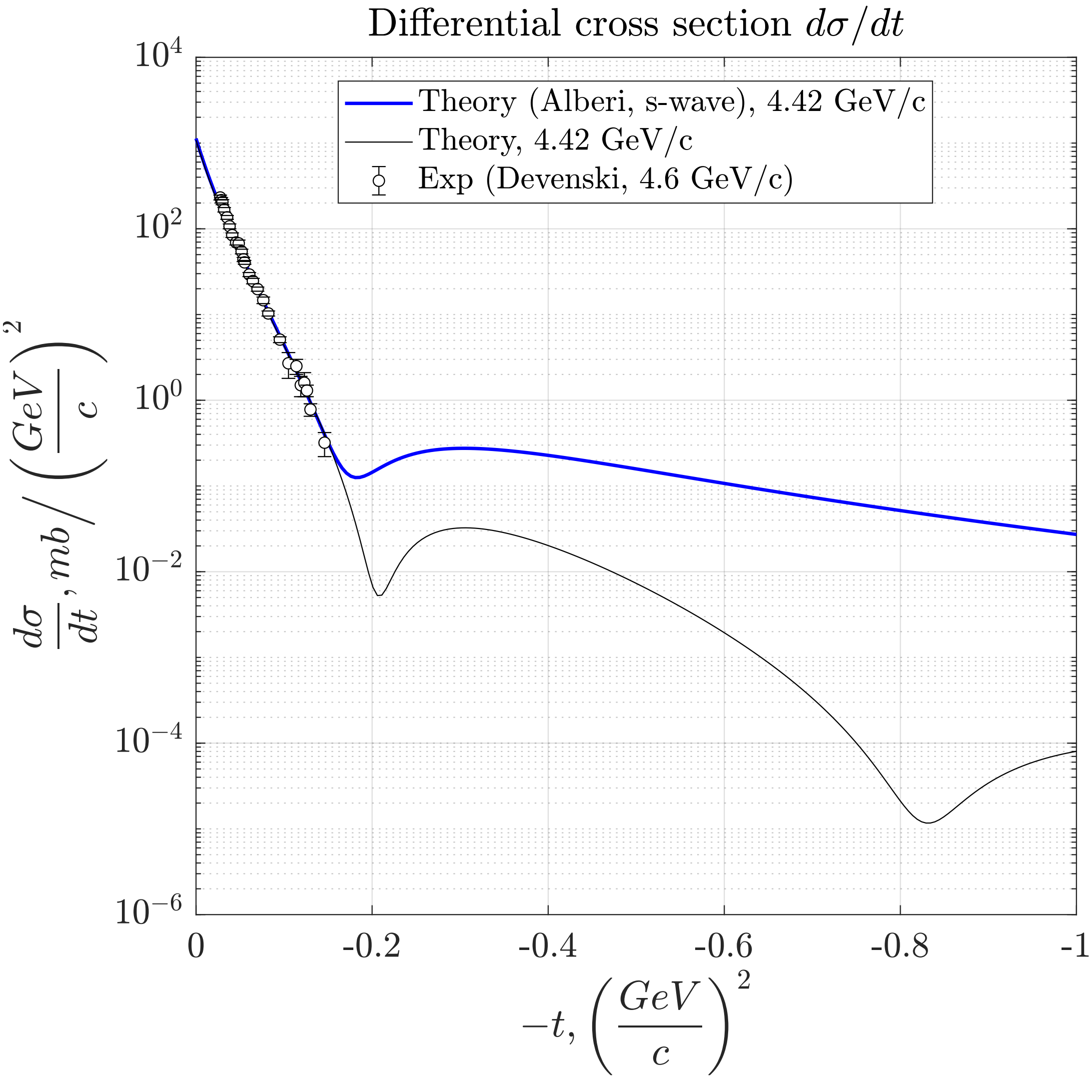

Figure 2: Differential cross section

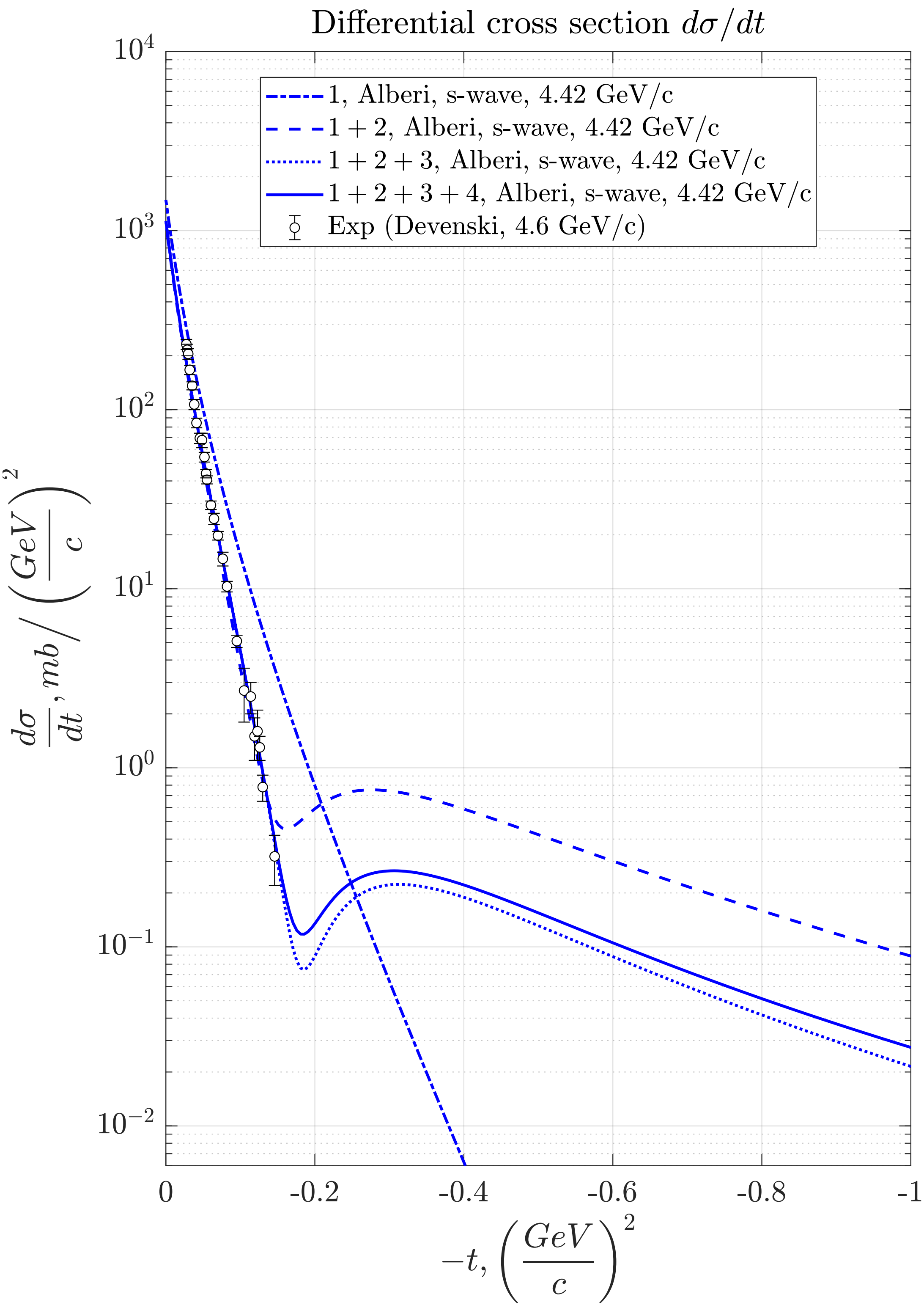

Figure 3: Differential cross section. Multiple scattering

Conclusion

As a result of the work, analytical expressions were obtained for the amplitude of deuteron-deuteron scattering ($dd \to dd$) in the formalism of the Glauber diffraction multiple scattering theory in the s-wave approximation: calculation in approximation of Nd-amplitudes (51) and calculation (71) in terms of nucleus – nucleus (approach of papers [6], [7], [11]).

The paper compared theoretical expressions for differential interaction cross sections with experimental data at energies of 4.42 GeV/c from the paper of Devensky [12]. A good description of the experimental data at small momentum transfers ($t < 0.15$ GeV/c) was obtained by both formulas (51), (71), obtained from different approaches. A comparison of the expressions (51) and (71) shows that there is a complete coincidence of the contribution of single scattering and a partial coincidence of analytical expressions of the contribution of double scattering. In addition, the diffraction cross section at zero transferred momentum $t = 0$ is the same. But in the region of transferred momentum t after the first minimum ($t > 0.15$ GeV/c) the Nd-amplitude approximation gives underestimated values of the differential cross section compared to (71), the discrepancy increases with increasing t. The value of the transmitted momentum itself, at which the first minimum is reached, also shifts towards a larger momentum. ($t \approx 0.21$ GeV/c for calculations using (51) and $t \approx 0.18$ GeV/c for calculations using (71)).

Despite these deviations, the formula (51), like the formula (71), is in agreement with the experimental data in the range $t < 0.15$ GeV/c.

Appendix

CD-Bonn model

We use the CD-Bonn model to describe the s-wave function of the deuteron. Parameters taken from the paper of Platonova M. N., Kukulin V. I. [10], parameters were obtained in the original paper Machleidt R. [9].

The s- and d- wave functions are represented as:

$$u(r) = r \sum_{i=1}^{5} C0_i \, e^{-A0_i r^2}, \quad w(r) = r^3 \sum_{k=1}^{5} C2_k \, e^{-A2_k r^2}$$

Table 2: Parameters of the deuteron wave function in the CD-Bonn model [9],[10]

i	$C0_i, \text{fm}^{-3/2}$	$A0_i, \text{fm}^{-2}$	$C2_i, \text{fm}^{-7/2}$	$A2_i, \text{fm}^{-2}$
1	0.2735080×10^{-1}	0.1248299×10^{-1}	0.1748384×10^{-3}	0.3229171×10^{-1}
2	0.2134582×10^{0}	0.1633824×10^{0}	0.1908814×10^{-1}	0.1911095×10^{0}
3	0.1044782×10^{0}	0.4765748×10^{-1}	0.2391706×10^{0}	0.8486950×10^{0}
4	0.2476300×10^{0}	0.5146949×10^{0}	-0.2814899×10^{0}	0.9955802×10^{1}
5	-0.2901292×10^{0}	0.2412550×10^{1}	-0.5504276×10^{0}	0.6477437×10^{2}

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