JOINT INSTITUTE FOR NUCLEAR RESEARCH
Bogoliubov Laboratory of Theoretical Physics

FINAL REPORT ON THE START PROGRAMME

Numerical analysis of the second-order radiative corrections to electron-positron annihilation process

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Participation period:
April 02 – May 13,
Winter Session 2023

Dubna, 2023

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Abstract

The main purpose of the work during the spring practice was to study the radiative corrections due to the initial state radiation in the process of electron-positron annihilation. The Fortran programming language was studied and the program code for numerical calculation of second-order radiative corrections was written. Also, the research papers by F.A. Berends, W.L. van Neerven and G.J.H. Burgers [1]; J. Blumlein, A. De Freitas, C.G. Raab, K. Schonwald [2] were studied.

1. Introduction

High energy e^+e^- colliders operating at large luminosity are crucial for measuring core parameters of the Standard Model with high precision and testing its structure. Previous experiments at LEP have obtained precise results on the parameters of the Z-boson. Future facilities such as ILC, CLIC, FCC-ee, and muon colliders are planned to operate at even higher energies and luminosities. This will allow for precise scans of the $t\bar{t}$ - threshold to measure properties of the top quark and produce the Higgs boson under clean conditions in ZH-final states to understand its properties in great detail.

However, highly precise measurements require exact knowledge of the QED radiative corrections for the $e^+e^- \rightarrow \gamma^*/Z^*$process, which must be known to two-loop order in the fine structure constant α, with additional logarithmic contributions in higher orders. During the spring practice, the first calculation of radiative corrections to the initial state O(α2) for this process was performed.

While the first calculation of the $O(\alpha^2)$ initial state radiative corrections was performed in Ref. [1], some approximations were made to simplify the integration process. However, the second calculation based on massive operator matrix elements (OMEs) showed deviations in the constant term at $O(\alpha^2)$, although the O(α) result and logarithmic terms at $O(\alpha^2)$ agreed. In order to determine which result is correct, a complete calculation of the scattering cross section without any approximation or assumption must be performed analytically.

In this paper, a thorough numerical calculation of the contributing terms is performed. The calculation also takes into account the axial-vector contributions and the differences in the components of the vector. Also, all terms up to $O((m_e^2/s)^0)$ are preserved. The final radiators can be expressed by Nielsen integrals:

$$S_{p,n} = \frac{(-1)^{n+p-1}}{(n-1)!p!} \int_0^1 \frac{dt}{t} \, ln^{n-1}(t) \, ln^p(1-zt) \tag{1}$$

$$S_{n-1,1}(x) = Li_n(x), \tag{2}$$

which cover the classical polylogarithms [4].
The radiator functions have the general structure

$$R\left(z, \alpha, \frac{s}{m^2}\right) = \delta(1-z) + \sum_{k=1}^{\infty} \left(\frac{\alpha}{4\pi}\right)^k C_k\left(z, \frac{s}{m^2}\right) \tag{3}$$

$$C_k\left(z,\frac{s}{m^2}\right) = \sum_{l=0}^{k} ln^{k-l}\left(\frac{s}{m^2}\right) c_{k,l}(z). \tag{4}$$

The respective differential cross sections are then given by

$$\frac{d\sigma_{e^+e^-}}{ds'} = \frac{1}{s}\sigma_{e^+e^-}(s')\, R\left(z,\alpha,\frac{s}{m^2}\right) \tag{5}$$

with $\sigma_{e^+e^-}(s')$ the scattering cross section without the initial state radiation (ISR) corrections and

$$z = \frac{s\prime}{s} \tag{6}$$

where s' is the invariant mass of the produced (off-shell) γ/Z boson. Here and in the following the mass m denotes the electron mass me, if not stated otherwise.

2 The process

Our focus is on the phenomenon of e^+e^- annihilation which leads to the creation of a virtual photon γ^*or virtual Z_0^* boson at an energy threshold of $s_0 \geq 4m_f^2$, where m_μ represents the mass of the muon and s denotes the squared cms energy of the annihilation process. Additionally, we can also explore the production of other fermionic final states such as $\tau^+\tau^-$, massless $q\bar{q}$, and their corresponding heavy quark pairs. The upper limit of the phase space for s_0 is defined as $s_0 \geq 4m_f^2$ .Under normal circumstances, we assume that $s_0 \geq 4m_f^2$ or alternatively, a more conservative cut. The differential Born cross section can be expressed as:

$$\frac{d\sigma_{e^+e^-}{}^{(0),\mathrm{I}}}{ds'} = \delta(s-s')\sigma^{(0)}(s') \tag{7}$$

where $\sigma^{(0)}(s')$ denotes the integrated cross section of one of the above processes. It corresponds to the annihilation diagram in Figure 1. For

s-channel e^+e^- annihilation into a virtual gauge boson (γ^*, Z^*)which decays into a fermion pair $f\bar{f}$, the scattering cross section reads

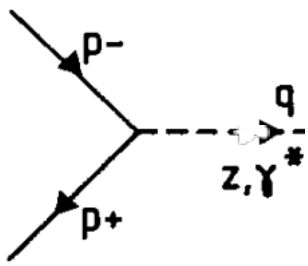

Figure 1: The Born cross section for the process $e^+e^- \rightarrow \gamma^*/Z_0^*$

$$\frac{d\sigma_{e^+e^-}}{d\Omega} = \frac{\alpha^2}{4s} N_{C,f} \sqrt{1 - \frac{4m_f}{s}} \times \left[\left(1 + cos^2\theta + \frac{4m_f^2}{s} sin^2\theta \right) G_1(s) - \frac{8m_f^2}{s} G_2(s) + \sqrt{1 - \frac{4m_f^2}{s}} cos^2\theta G_3(s) \right] \quad (8)$$

$$\sigma^0(s) = \frac{4\pi^2}{3s} N_{C,f} \sqrt{1 - \frac{4m_f}{s}} \left[\left(1 + \frac{2m_f^2}{s} \right) G_1(s) - 6\frac{6m_f^2}{s} G_2(s) \right] g(s) \, , \quad (9)$$

In the current work, in order to obtain the Born cross-section in the s-channel, it is assumed that the fermions of the final state are not electrons. The electron mass is disregarded in Eqs. (8, 9) kinematically. The fine structure constant is represented by α, while $N_{C,f}$ corresponds to the number of colors of final state fermions. The value of $N_{C,f} = 1$ for colorless fermions and $N_{C,f}$ =3 for quarks. The function $g(s)$ is set to 1 for pure perturbative calculations. The variables used in the study are s for the cms energy, Ω for the spherical angle, Ω for the cms scattering angle, and $G_1(s)|_{i=1...3}$ as the effective couplings.

$$G_1(s) = Q_e^2 Q_f^2 + 2Q_e Q_f v_e v_f \Re[\chi_Z(s)] + \left({v_f}^2 + {a_f}^2\right)^2 ({v_e}^2 + {a_e}^2)^2 |\chi_Z(s)|^2$$
$$G_2(s) = ({v_e}^2 + {a_e}^2)^2 {a_f}^2 |\chi_Z(s)|^2$$
$$G_3(s) = 2Q_e Q_f a_e a_f \Re[\chi_Z(s)] + 4v_e v_f a_e a_f |\chi_Z(s)|^2$$

The reduced Z–propagator is given by

$$\chi_Z(s) = \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z} \quad (10)$$

where M_Z and Γ_Z are the mass and the width of the Z-boson and m_f is the mass of the final state fermion. $Q_{e,f}$ are the electromagnetic charges of the electron ($Q_e = -1$) and the final state fermion, respectively, and the electro–weak couplings v_i and a_i are given by

$$v_e = \frac{1}{sin\theta_w cos\theta_w} \left[I_{w,e}^2 - 2Q_e sin^2\theta_w \right], \quad (11)$$

$$a_e = \frac{1}{sin\theta_w cos\theta_w} I_{w,e}^3, \quad (12)$$

$$v_f = \frac{1}{sin\theta_w cos\theta_w} \left[I_{w,f}^2 - 2Q_f sin^2\theta_w \right], \quad (13)$$

$$a_f = \frac{1}{sin\theta_w cos\theta_w} I_{w,f}^3, \quad (14)$$

where θ_w is the weak mixing angle, and $I_{w,e}^3 = \pm$½ the third component of the weak isospin for up and down particles, respectively.

For the radiative corrections studied below, we will consider the integrated cross section (9) in the energy region of the Z–peak.

In the following we will use the fine structure constant with the normalization

$$a = \frac{\alpha}{4\pi}\,. \qquad (15)$$

The scattering cross section including the contributions due to initial state radiation can be expressed as follows

$$\sigma(s) = \int_0^1 dz\, R(z;\alpha;L)\sigma_0(zs), \qquad (16)$$

where $R(z;\alpha;L)$ is the distribution–valued [5] radiative function, with

$$L = ln\left(\frac{s}{m_e^2}\right) \qquad (17)$$

The different radiators calculated in the present paper sum to the following distribution

$$R(z;\alpha;L) = a_1 R_1^{\gamma}(z,L) + a^2 \qquad (18)$$

3. Program code for calculating radiative corrections

Important fragments of the program for numerical calculation of radiative corrections will be shown below. The work of F.A. Berends, W.L. van Neerven and G.J.H. Burgers [1] was taken as a basis. The Fortran language was chosen as the tool for the following reasons:

1) Fortran is a standard programming language for scientific computing, which is widely used in high-energy physics. This means that there are many libraries and tools that can be used to solve problems in this area.

2) Fortran has a high speed of program execution, which is especially important for complex computational tasks related to radiative corrections. This allows you to get results faster and use computer resources more efficiently.

3) Fortran has powerful capabilities for working with matrices and data arrays, which is often used in numerical methods used for the analysis of radiative corrections. This allows you to process large amounts of data quickly and efficiently.

```
C...  HIGHER ORDER ISR CORRECTIONS TO E+E- ANNIHILATION
      IMPLICIT REAL*8(A-H,O-Z)
      INTEGER IOALPN,IOALPX,IOLN,IOLX
      COMMON /PCONST/ AME,ALPHA,AMZ,AMTL,AITL,AIE,SW2,GAMZ,QE,QTL,NCF
     & ,CONV
      COMMON /MCONST/ PI,DZ2,DZ3,DZ4
      COMMON /PARAMS/ S,ZMIN,ALLG,DELTA
      COMMON /EWPARAMS/ SW,VE,AE,VF,AF,CW
      COMMON /FLAGS/ IOALPN,IOALPX,INLO
      EXTERNAL SIGR,SIGMA0

      CALL RADINIT(DUMMY)
      CALL ELWEPAR(EWC)
C...  USER PARAMETERS:
      EBEAM = AMZ/2D0 - 0.1D0   ! BEAM ENERGY, GeV
      ZMIN = 0.1D0               ! MINIMAL Z VALUE
      PRINT *,'EBEAM=',EBEAM
      PRINT *,'ZMIN =',ZMIN
C... ORDER IN ALPHA (IALPN.LE.IALP.LE.IALPX)
      IOALPN= 2
      IOALPX= 2
C... NLO TYPE (INLO=0,1,2: LO,NLO,NNLO)
      INLO = 2

C...  AUXILIARY PARAMETER (SOFT-HARD SEPARATOR)
      DELTA = 1D-5

C...  PARAMETERS FOR INTEGRATION PRECISION
      REPS  = 1D-6
      RAPS  = REPS*REPS

C...  PREPARATIONS
      S = 4D0*EBEAM**2
      ALLG = DLOG(S/AME**2)
      PRINT *,'ALLG =',ALLG

C.... BORN CROSS SECTION
      S0 = S
C     SIG0 = SIGMA00(S0)
      SIG0 = SIGMA0(S0)

C.... CORRECTION CROSS SECTION: SOFT+VIRTUAL EMISSION
      CORSV = SIGMA0(S0)*RADSV(DUMMY)
      PRINT *, ' CORSV= ', CORSV

C.... CORRECTED CROSS SECTION: HARD EMISSION
      ZN  = ZMIN
      ZX  = 1D0 - DELTA
      ZST = (ZX-ZN)/4D0
      CALL SIMPS(ZN,ZX,ZST,REPS,RAPS,SIGR,ZZZ,RSZ,RES2,RES3)
      CORH = RSZ

      PRINT *, ' CORH = ', CORH

      CORR = CORH + CORSV
      SIG1 = SIG0 + CORR

      PRINT *,'BORN=',SIG0
      PRINT *,'SIG1=',SIG1
      PRINT *,'CORR=',CORR
      PRINT 77,CORR/SIG0*1D2
 77   FORMAT('RELATIVE CORRECTION %=',1F12.5)
```

Figure 2: Initialization of functions, constants and parameters

```
C-----------------------------------------------------------
      REAL*8 FUNCTION RADH(Z)
C...  RADIATOR: HARD EMISSION
      IMPLICIT REAL*8(A-H,O-Z)
      INTEGER IOALPN,IOALPX,INLO
      INTEGER IOALP,IOL
      COMMON /FLAGS/ IOALPN,IOALPX,INLO
      COMMON /PCONST/ AME,ALPHA,AMZ,AMTL,AITL,AIE,SW2,GAMZ,QE,QTL,NCF
     & ,CONV
      COMMON /MCONST/ PI,DZ2,DZ3,DZ4
      COMMON /PARAMS/ S,ZMIN,ALLG,DELTA
      COMMON /EWPARAMS/ SW,VE,AE,VF,AF,CW
      EXTERNAL CHARD

      RADH = 0D0
      DO IOALP=IOALPN,IOALPX
        DO IOL=IOALP-INLO,IOALP
          RADH = RADH + CHARD(Z,IOALP,IOL)*(ALPHA/PI)**IOALP*ALLG**IOL
        ENDDO
      ENDDO

C...  PRINT *, 'radh= ' , RADH
      RETURN
      END

C-----------------------------------------------------------
```

Figure 3: Contribution of hard-photon emission

```
      REAL*8 FUNCTION RADSV(DUMMY)
C...  RADIATOR: SOFT+VIRTUAL EMISSION
      IMPLICIT REAL*8(A-H,O-Z)
      INTEGER IOALPN,IOALPX,INLO
      COMMON /FLAGS/ IOALPN,IOALPX,INLO
      COMMON /PCONST/ AME,ALPHA,AMZ,AMTL,AITL,AIE,SW2,GAMZ,QE,QTL,NCF
     & ,CONV
      COMMON /MCONST/ PI,DZ2,DZ3,DZ4
      COMMON /PARAMS/ S,ZMIN,ALLG,DELTA
      COMMON /EWPARAMS/ SW,VE,AE,VF,AF,CW
      EXTERNAL CSV

      RASV = 0D0
      DO IOALP=IOALPN,IOALPX
        DO IOL=IOALP-INLO,IOALP
          RASV = RASV + CSV(IOALP,IOL)*(ALPHA/PI)**IOALP*ALLG**IOL
        ENDDO
      ENDDO

C...  PRINT *, 'radsv= ' , RADSV
      RETURN
      END
C-----------------------------------------------------------
```

Figure 4: Contribution of soft and virtual photon emission

Conclusions

During the spring internship the second-order radiative corrections to the process of $e^+e^- \to \gamma^*/Z^*$ were studied in detail and the Fortran programming language was mastered. Mastering this language allows you to develop and optimize programs to solve complex mathematical problems, as well as manage large amounts of data.

The study of radiative corrections in the process of electron-positron pair formation is important for an accurate description of physical phenomena at the elementary particle level. These corrections can significantly influence the measured cross sections and distributions of particles in the final state, which should be taken into account when analyzing experimental data and verifying theoretical models. In addition, the study of radiative corrections can lead to more accurate determinations of fundamental parameters, such as masses and properties of elementary particles.

Also, a program was developed to numerically analyze the contribution to the radiative corrections of the 2nd order to the initial state radiation (ISR) of positron-electron annihilation.

In the future, higher order contributions are planned to be added to the program. In the near future I plan to continue working on this topic, which will be the basis of my future master's and PhD thesis under the guidance of my supervisor for spring practice - Andrej Borisovich Arbuzov.

Acknowledgements

In the end, I would like to thank: my supervisor — Andrej Borisovich Arbuzov for his active mentoring throughout our joint work, the leadership of the Jointinstitute for nuclear research for financial support; Elena Gennadievna Karpova, who provided prompt feedback throughout the entire time from the moment the application was submitted.

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