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FINAL REPORT ON THE
SUMMER STUDENT PROGRAM

Study of plasma waves in highly dissipative systems of Josephson junctions

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Abstract—***We investigate the Current Voltage characteristics (CVC) and generation of longitudinal plasma wave in arrays of coupled Josephson junctions with large dissipation, in order to obtain new qualitative phenomena caused by coupling-dissipation correlation. We discover the parturition of the second hysteresis zone on the IV characteristic and generation of the plasma wave caused by collective state generation in Josephson arrays. All simulations are made using CCJJ+DC model. The possible applications of discovered effect have been discussed.***

Keywords—Josephson Junctions; nanostructures; collective phenomena; superconductivity.

1. Introduction

The Josephson junctions are intensely studied nanostructures based on high-T_C superconductors and topological insulators. The superconductivity phenomena in arrays of Josephson junctions (JJs) occurs because of tunneling of Cooper pairs trough the insulator layers.

The Josephson nanostructures have multiple applications in THz emitters, metrology and nanoelectronics. Current vast theoretical and experimental studies on c-axis transport in superconducting heterostructures have detected multiple-branch structures in the CVCs [1], microwave resonant absorptions [2,3] and transitory effects created by current injection. To describe such an interesting structure, a system of nonlinear equations is used, and it might be solved only numerically. In [4] the capacitively coupled Josephson junction (CCJJ) model was elaborated to describe the inner characteristics of JJs. Diffusion current plays also a significant role in the charge asymmetry effect in the array of JJs, as was shown in [5,6].

In the work [7] was studied the influence of dissipation and coupling parameters on the behavior of arrays of Josephson Junction with week dissipation and strong coupling. We extend our research on the systems with large dissipation and strong coupling by numerical simulations using CCJJ+DC model.

2. THE CCJJ+DC MODEL

To reproduce the Current Voltage characteristics we research the phase dynamics of the system in the framework of the capacitively coupled Josephson junctions with diffusion current (CCJJ+DC) model [5,6], which is described by the system of nonlinear equations:

$$dV_l/ dt=I- \sin\varphi_l -\beta d\varphi_l /dt \qquad (1)$$

$$d\varphi_l /dt=V_l- \alpha(V_{l+1}+V_{l-1}-2V_l) \qquad (2)$$

where V_l is the voltage between the superconducting nanolayers, φ_l is the phase difference between (l+1)th and lth layers, and α represents coupling parameter between the JJs and is determined only by the material type, and β is dissipation parameter, which can be varied with temperature. We take into account the periodic boundary conditions in this work.

To calculate the voltages $V_l(I)$ at each value of bias current, we reproduce the dynamics of the system by numerically solving the system of equations in Eq. (1,2) [8] by implementing the fourth-order Runge-Kutta method. Therefore, at a locked value of bias current we obtain the voltage time-dependences in each junction. As a consequence, we can estimate the charge time-dependences in each layer as the voltage difference in the successive junctions.

The current value is heightened or declined by a small amount of bias current step after finishing the computation for bias current to determine the voltages in all junctions at the following point of the CVC. The charge density Q_l, and in the text we will name it charge, in the superconducting layer l is linearly dependent on the difference between the voltages on the successive topological insulating layers. The particulars of the simulation model are given in [9].

To proove the chaotic behavior in the system we calculate Lyapunov exponent for each current value by the formula:

$$LE_j=1/T_p \ln(||\mathbf{d}_{j+1}||/||\mathbf{d}_j||) \qquad (3)$$

where $||\mathbf{d}_j||=[\Sigma_l^N{}_{=1}(\Delta\varphi_l(t_j))^2+\Sigma_l^N{}_{=1}(\Delta V_l(t_j))^2]^{1/2}$.

3. STUDY OF THE SECOND HYSTERESIS ZONE

The simplest way to obtain information about systems of Josephson junctions is to study their current-voltage characteristics by varying its intrinsic parameters. In our work we vary couping parameter α from 0 to 1.3 and dissipation parameter β from 0.2 (the obtained results are in agreement with Ref. [5]) to 1.3.

In the Figure 1 is the variation of CVC with the increase of dissipation parameter (from 0.2 to 1.3) for constant coupling parameter (α=1) and number of junctions (N=10). With increase of dissipation was observed the reduction of McCumber hysteresis zone, and for threshold dissipation parameter value (around 0.7) appears a new qualitative phenomena: the second hysteresis zone on the CVC characteristic.

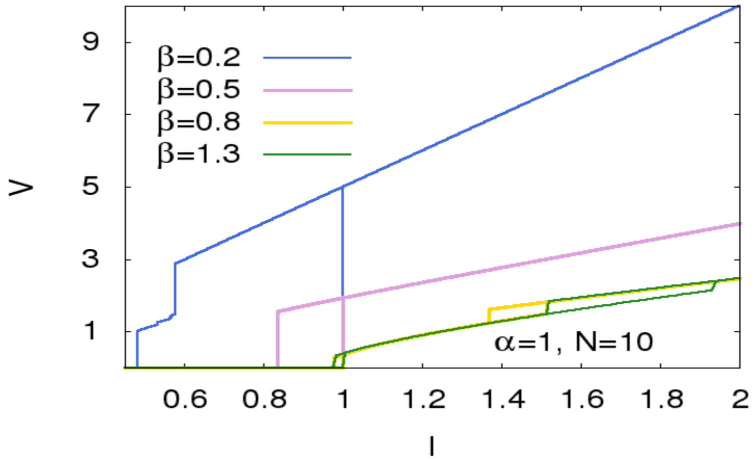

Fig. 1. The variation of CVC with increasing of β from 0.2 to 1.3 for a fixed coupling parameter α=1, and number of junctions in array N=10.

Appearance of the second hysteresis zone is more visible in case of successive variation of coupling parameter from 0 to 1.3 for a fixed dissipation value (Figure 2). The second hysteresis zone appears for coupling parameter value of α=0.501. Near the threshold value CVC has chaotic elements.

The existence of the second hysteresis zone offers a new perspective of implementation of such systems in nanoelectronics, for example allows us to design a multi-state logical circuits based on arrays of Josephson junctions.

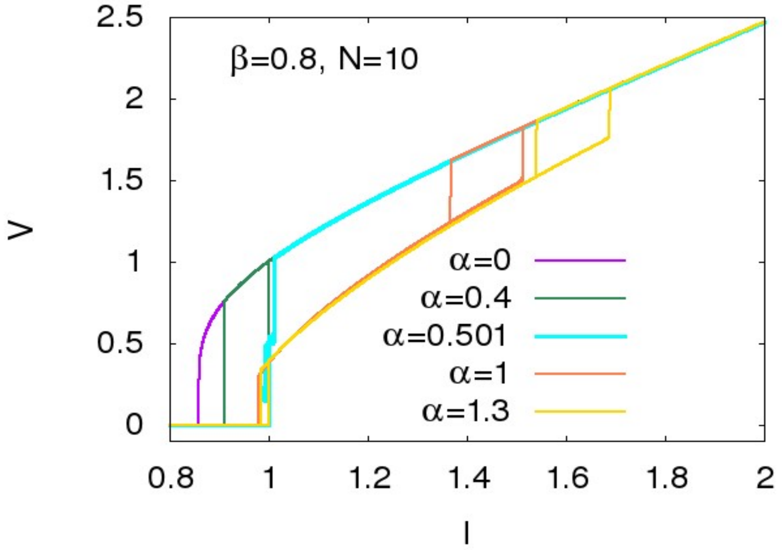

Fig. 2. Appearance of the second hysteresis zone on CVC with increase of α from 0 to 1.3 for β=0.8 and N=10.

In Figure 2 a we can clearly see that second hysteresis zone appears, the system has signs of chaotic behavior. In addition is presented the Current Voltage characteristic for each junction and distinctly can be seen brunching, that does not occur for the same value of dissipation parameter for higher values of coupling parameter.

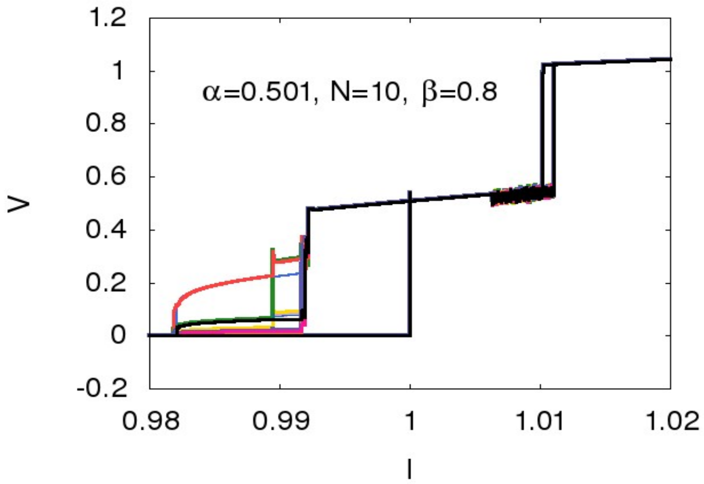

Fig. 2a. CVCs for N=10, β=0.8 and α=0.501 for different junctions in stack

4. PROOF OF THE COLLECTIVE BEHAVIOR

The appearance of the second hysteresis zone on current-voltage characteristic is caused by complex phenomena that occur inside of the stack of Josephson junctions. To study those phenomena we analyze the short-time dynamics of charge distribution on the layers of the JJs array.

The simulation was effectuated for increasing (Fig. 3) and decreasing (Fig. 4) current. Was observed generation of the c-axis charge traveling wave for the current values that correspond to the second hysteresis zone. More detailed for both situations (the increase and the decrease of current), as is seen in Figures 3 and 4, we can observe that the longitudinal plasma wave spreads from the McCumber hysteresis zone to the second hysteresis zone.

However, with the appearance of second hysteresis zone not only lessens the McCumber hysteresis zone, but also disappears the branching in that area.

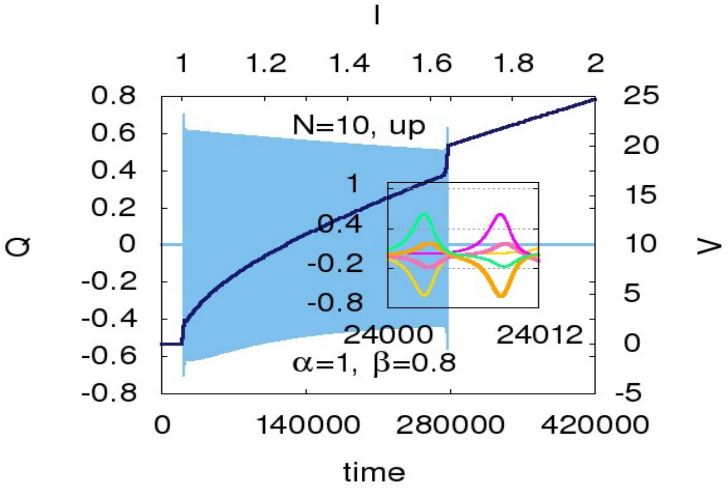

Fig. 3. The charge-time dependence for the increase of current. The second hysteresis zone is characterized by significant charge oscillations.

In Figures 3 and 4 with darker color is presented the Current-Voltage characteristic and with a lighter color is presented the temporal dependence of charge. On the insets is presented the zoomed dynamics of charge distribution for two different layers.

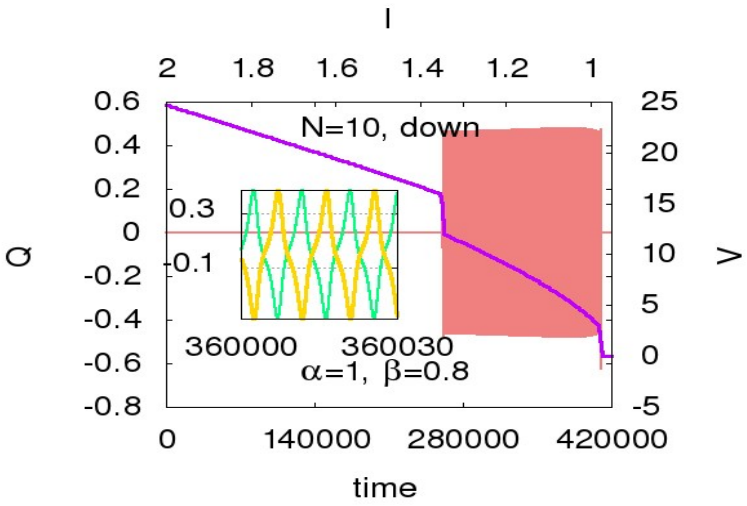

Fig. 4. The charge-time dependence for the decrease of current. The second hysteresis zone is characterized by significant charge oscillations.

For a fixed value of large-enough dissipation parameter in Figure 2 we observed the appearance of second hysteresis zone for a large-enough coupling parameter (α=0.501). Therefore, it is a phenomenon that depends on the coupling between the junctions, and consequently it can be considered a collective state of junctions. It can occur only in an array with numerous junctions and is strictly dependent on the material properties of the superconductor and topological insulator.

In addition, the appearance of second hysteresis zone is connected to the temperature, because the dissipation parameter plays an important role in it. The second hysteresis zone appears only for the large-enough dissipation parameters when dissipation effects are contra-balanced by nonlinearity.

We can also observe that the system emits THz electromagnetic radiation until the collapse of the outermost back to traveling wave branch and that creation of LPW leads to absence of the emission. Furthermore, we can also observe that the decrease in amplitude of the charge oscillations is caused by the appearance of THz emission right after the second hysteresis zone.

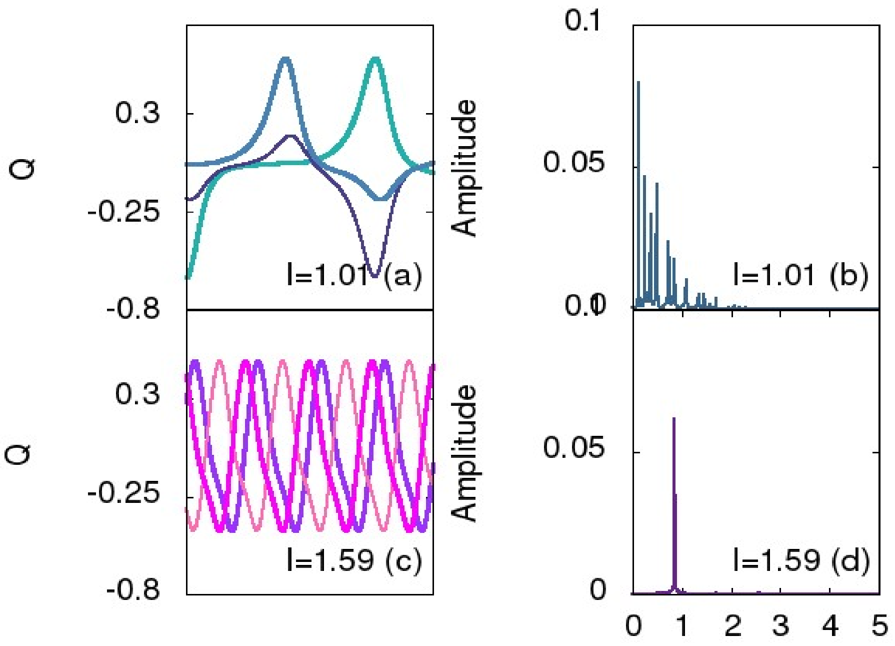

Fig. 5. Charge oscillations (a) and (c) with the corresponding fast Fourier transforms (b) and (d) for the increase of current for corresponding values of current

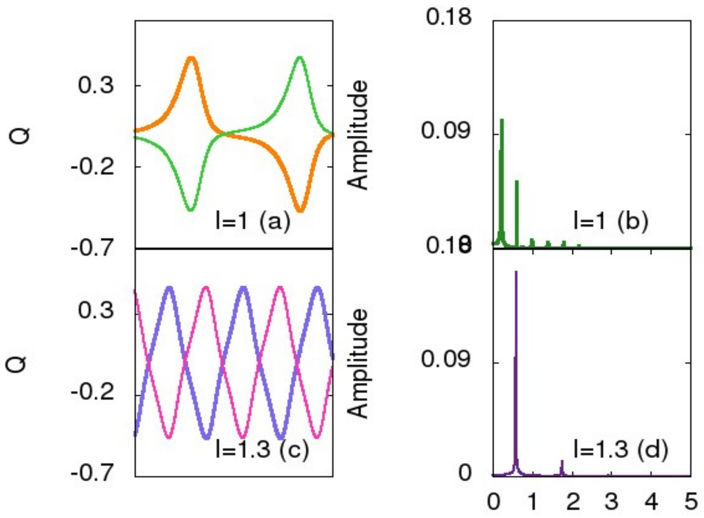

Fig. 6. Charge oscillations (a) and (c) with the corresponding fast Fourier transforms (b) and (d) for the decrease of current for corresponding values of current

The detailed study of the shape of charge oscillations for increasing and decreasing of current was elaborated during this work. We investigate fast Fourier transform and charge correlations between consecutive layers of the JJ arrays for different current values.

Was observed that the form of oscillations has complex and more unharmonious structure near the McCummber zone and becomes almost harmonic near the end of the second hysteresis zone. It can be easily seen on the fast Fourier transforms attached to the charge oscillations for corresponding currents for either decreasing or increasing current (Figures 5 and 6).

The fact that the oscillations become more harmonious toward the second hysteresis zone and after reaching the second hysteresis zone disappear leads us to the hypothesis that this phenomenon might be connected to the solitons.

Suddenly, the outermost branch collapses back to the TW branch, forming the second hysteresis zone. The switch to the TW branch is followed by the creation of LPW, which evolves till the end of McCumber hysteresis zone. As is evident from the inset of figure 4, we observe the display of traveling LPW. On the inset are shown charge oscillations for the first and second superconducting layers.

5. STUDY OF CHARGE OSCILLATIONS IN CHAOTIC SYSTEMS

The appearance of chaotic behavior can be detected by the study of the shape of charge temporal dependence, by charge correlations on the consequent layers, and by the study of Lyapunov exponent.

The simulation was run for the transition value of coupling parameter of $\alpha=0.501$, dissipation parameter $\beta=0.8$ and number of junctions in stack of 10. From Figure 3 can be observed that the current-voltage characteristic is highly influenced by the shape of the charge oscillations. Also, on the inset we can see the zoomed in temporal charge oscillations which are clearly unharmonious.

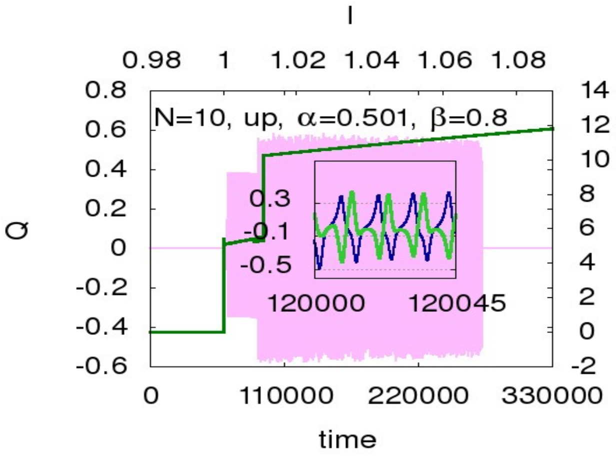

Fig. 3. The charge-time dependence for increase of current

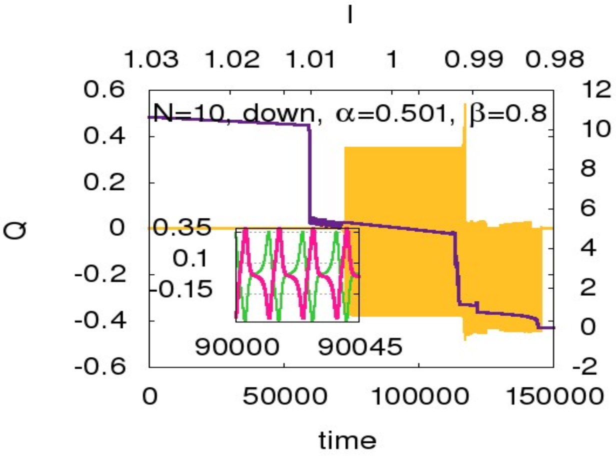

Fig. 4. The charge-time dependence for decrease of current

Similar to the increase of current, for the decrease of current we can see some resemblance between the shape of IV-characteristic with the phase dynamics of the system. However, the temporal charge oscillations are highly unharmonious.

To prove the unharmonious behavior of the system were contrived the charge correlations on the consecutive layers of the topological insulator for both the decrease and the increase of current, which are presented in Figures 5 and 6. In the insets are presented corresponding temporal charge dependences for corresponding values of current.

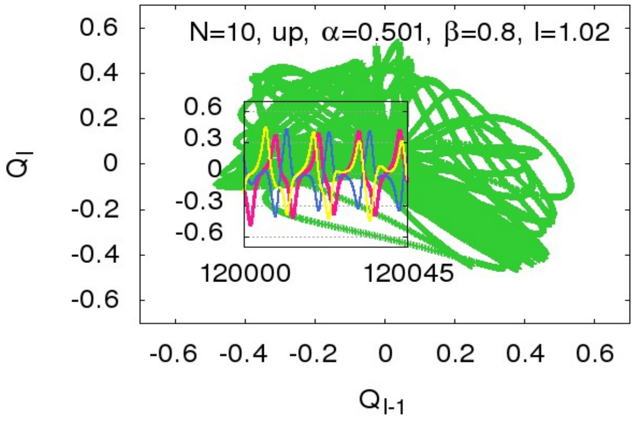

Fig. 5. Charge correlations Lissajous diagrams for the successive layers for the system of N=10, α=0.501, and β=0.8 for the increase of current

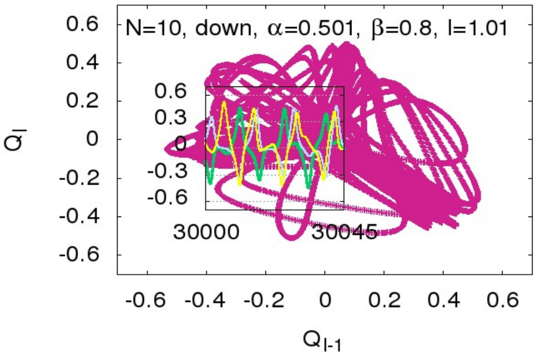

Fig. 6. Charge-charge Lissajous diagrams for the successive layers for the system of N=10, α=0.501, and β=0.8 for the decrease of current

Were made the Lissajous charge-charge correlations for the chaotic regions and, as can be evident from the Figures 5 and 6, for the chaotic region all of the correlations are lost. We have chosen two random values of current, in case of the increase of current I=1.02, and in case of decrease of current, I =1.01.

5. LYAPUNOV EXPONENT

In this work we calculate maximal Lyapunov exponent to prove the chaotic behavior of the system. The commonplace plan is to follow two adjacent paths and to estimate their mean logarithmic rate of partition. When the two paths become too far distant, one of them is ought to be moved backward to the proximity of the other, along the line of partition. A conservative method is to implement this procedure at every iteration.

As one can see in Figure 7, there are two distinct regions for the Lyapunov exponent, one of them is for LE>0, the other one is for LE=0, and the third one is for LE<0. We can view the region with LE>0 as a strictly chaotic one, and for LE=0 the system is in marginal stability state. The second hysteresis zone also did not form correspondingly just yet, the region being really thin.

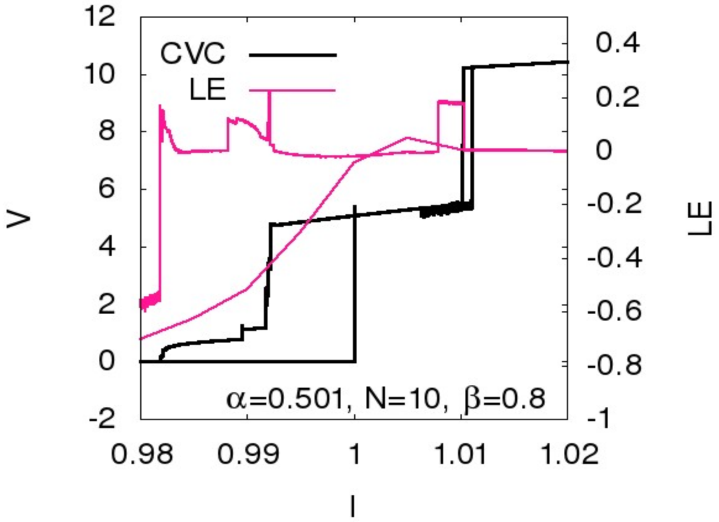

Fig. 7. Lyapunov exponent (pink line) as function of current and CVC for the transitory state of N=10 junctions, α=0.501, and β=0.8

6. CONCLUSIONS

In summary, we predict the appearance of second hysteresis region in the current-voltage characteristics in the system of Josephson junctions with numerous junctions in stack, high dissipation and strong coupling. These results can be obtained for layered high T_c superconductors, as well. We establish the correspondence of the characteristics of this structure with the traveling longitudinal plasma wave on the superconducting layers. We observe that the second hysteresis zone is formed by traveling wave and outermost branches in the current-voltage characteristics. The discovered phenomenon might be used in quantum computers, terahertz oscillators,traveling wave parametric amplifiers and mixers. It is our hope that this theoretical study could prompt an experimental validation of the existence of second hysteresis zone in the current-voltage characteristics of discussed systems in this report. Was detected and studied the transitory state from local to collective in Josephson junctions by Lyapunov's exponent, charge-charge Lissajous correlations, by phase dynamics of the system, and by Current-Voltage characteristics. The transitory state was found to be chaotic.

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Note: The best results wasn't included in this report, and will be published in scientific paper.

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