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Flerov laboratory of nuclear reactions

FINAL REPORT ON THE

SUMMER STUDENT PROGRAM

Resonance scattering of protons on the ^{8}He and ^{6}He

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Abstract

This work is devoted to a theoretical study of the reaction of resonant scattering of protons by nuclei ^{8}He and ^{6}He. To study proton scattering on the nucleus ^{8}He the two-channel approximation of the strong coupling of channels is used. Main assumption is isospin symmetry of nuclear interaction, so the interaction between clusters can be represented as a sum of isospin pure terms.

To study proton scattering on the nucleus ^{6}He we use a more advanced, five-channel approach. The systems ^{6}Li and ^{6}He are considered as three-body cluster form α-N-N. The the interaction of the nucleon with such a system was divided into the interaction with the α-particle and with two external nucleons. For our calculations, we used several of the most modern phenomenological nucleon-nucleon potentials, including spin and isospin exchange operators.

Two-chanel model

In the framework of this model, a two-channel scattering problem with the following channels is considered:

1. $^{8}He(0^{+},2)+p$
2. $^{8}Li^{*}(0^{+},2)+n$

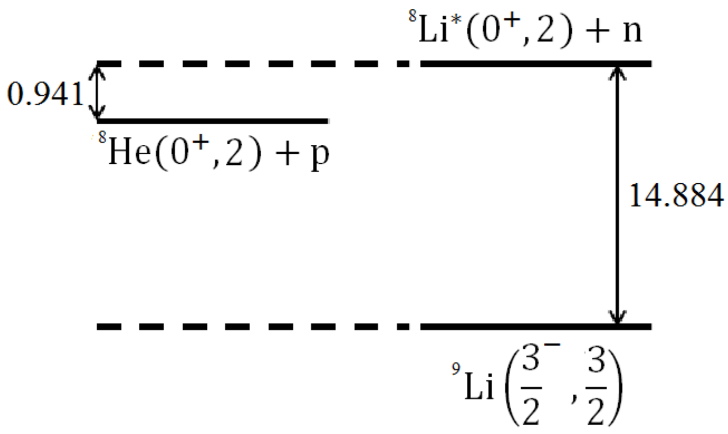

Fig. 1. Two-channel scattering scheme.

Consider the following wave functions in a cluster representation: $\Psi_{^{8}He+p} \equiv |\,^{8}He+p\rangle$ and $\Psi_{^{8}Li+n} \equiv |\,^{8}Li^{*}+n\rangle$. Using wave functions with a certain isospin $\Psi_{T,T_3} \equiv |T,T_3\rangle$, we can rewrite these wave functions in the following form:

$$|\,^{8}He+p\rangle = C^{3/2\,-3/2}_{2\,-2\,1/2\,1/2}|3/2\,,-3/2\rangle + C^{5/2\,-3/2}_{2\,-2\,1/2\,1/2}|5/2\,,-3/2\rangle$$

$$|\,^{8}Li^{*}+n\rangle = C^{3/2\,-3/2}_{2\,-1\,1/2-1/2}|3/2\,,-3/2\rangle + C^{5/2\,-3/2}_{2\,-1\,1/2-1/2}|5/2\,,-3/2\rangle$$

That is:

$$|\,^{8}He+p\rangle = -\sqrt{\frac{4}{5}}|3/2\,,-3/2\rangle + \sqrt{\frac{1}{5}}|5/2\,,-3/2\rangle$$

$$|\,^{8}Li^{*}+n\rangle = \sqrt{\frac{1}{5}}|3/2\,,-3/2\rangle + \sqrt{\frac{4}{3}}|5/2\,,-1/2\rangle$$

On the other hand:

$$|3/2\,,-3/2\rangle = -\sqrt{\frac{4}{5}}|\,^{8}He+p\rangle + \sqrt{\frac{1}{5}}|\,^{8}Li^{*}+n\rangle$$

$$|5/2\,,-3/2\rangle = \sqrt{\frac{1}{5}}|\,^{8}He+p\rangle + \sqrt{\frac{4}{5}}|\,^{8}Li^{*}+n\rangle$$

Main assumption is isospin symmetry of nuclear interaction, so the interaction between clusters can be represented as a sum of isospin pure terms:

$\hat{V} = \hat{P}_{3/2}V_{3/2} + \hat{P}_{5/2}V_{5/2}$, where projection operators can be written as:

$\hat{P}_{3/2} = |3/2\ , -3/2\rangle\langle 3/2\ , -3/2|$

$\hat{P}_{5/2} = |5/2\ , -3/2\rangle\langle 5/2\ , -3/2|$

and their matrix elements are:

$\langle\, ^8Li^* + n|\hat{P}_{3/2}|\, ^8Li^* + n\rangle = 1/5$ $\qquad$ $\langle\, ^8Li^* + n|\hat{P}_{5/2}|\, ^8Li^* + n\rangle = 4/5$

$\langle\, ^8Li^* + n|\hat{P}_{3/2}|\, ^8He + p\rangle = -2/5$ $\qquad$ $\langle\, ^8Li^* + n|\hat{P}_{5/2}|\, ^8He + p\rangle = 2/5$

$\langle\, ^8He + p|\hat{P}_{3/2}|\, ^8He + p\rangle = 4/5$ $\qquad$ $\langle\, ^8He + p|\hat{P}_{5/2}|\, ^8He + p\rangle = 1/5$

So, from Schroedinger equation we get a system of two coupled equations:

$$\begin{cases} \left(T - E + V_{coul} + \frac{4V_{3/2}+V_{5/2}}{5}\right)\Psi_{\,^8He+p} + \frac{2}{5}\left(V_{5/2} - V_{3/2}\right)\Psi_{\,^8Li+n} = 0 \\ \left(T - (E - 0.941) + \frac{V_{1/2}+2V_{3/2}}{5}\right)\Psi_{\,^8Li+n} + \frac{2}{5}\left(V_{5/2} - V_{3/2}\right)\Psi_{\,^8He+p} = 0 \end{cases} \quad (1)$$

where E is energy in the $^8He(0^+, 2) + p$ channel.

The solutions of the system (1) can be obtained for different parameters $V_{5/2}$ and $V_{3/2}$. Consider the most obvious boundary cases.

1) $V_{5/2} = V_{3/2}$.

In this case, the system of coupled equations (1) turns into a system of unrelated equations. We observe a pure channel function $\Psi_{\,^8He+p}$, mixed by isospin in initial proportions.

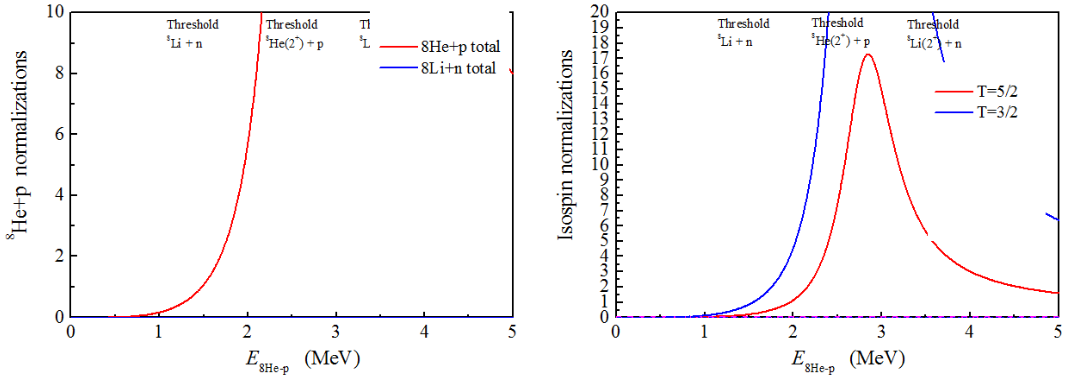

Fig. 2. The case of $V_{5/2} = V_{3/2}$.

2) $V_{3/2} = 0$.

In this case, we see mixed channel functions, but isospin becomes pure $T = 5/2$.

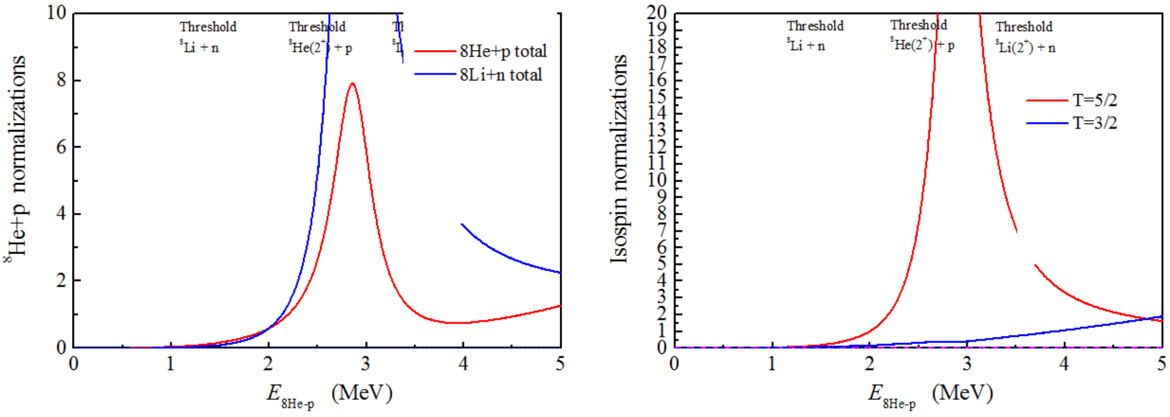

Fig. 3. The case of $V_{3/2} = 0$.

3) $V_{5/2} = 0$.

Similarly to case 2), we observe mixed channel functions, and the isospin becomes pure T = 3/2.

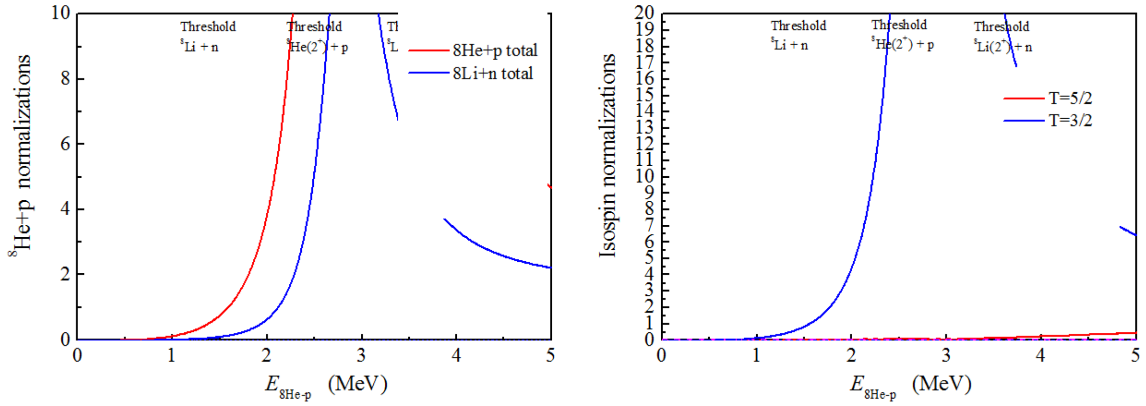

Fig. 4. The case of $V_{5/2} = 0$.

Five-channel model

In the framework of this model, a five-channel scattering problem with the following channels is considered:

1. $^6Li(1^+, 0) + n$
2. $^6He(0^+, 1) + p$
3. $^6Li^*(0^+, 1) + n$
4. $^6He^*(2^+, 1) + p$
5. $^6Li^*(2^+, 1) + n$

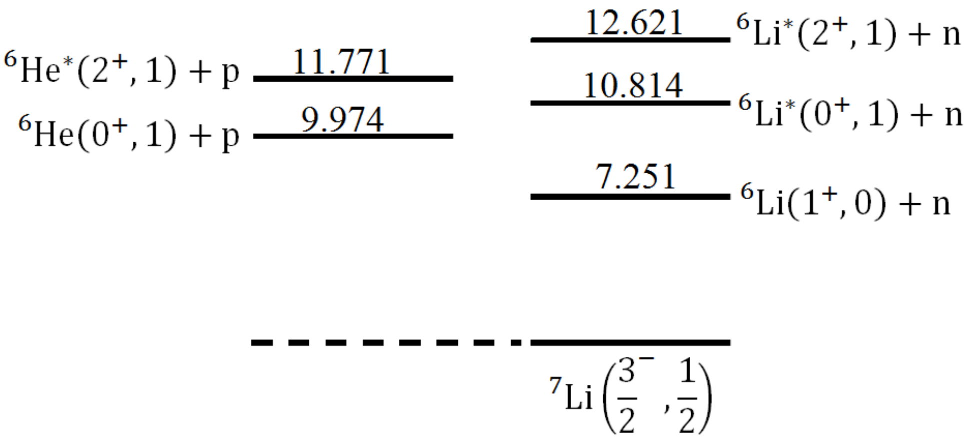

Fig. 5. Two-channel scattering scheme.

The systems ^6Li and ^6He can be represented with good accuracy in the cluster form α-N-N.

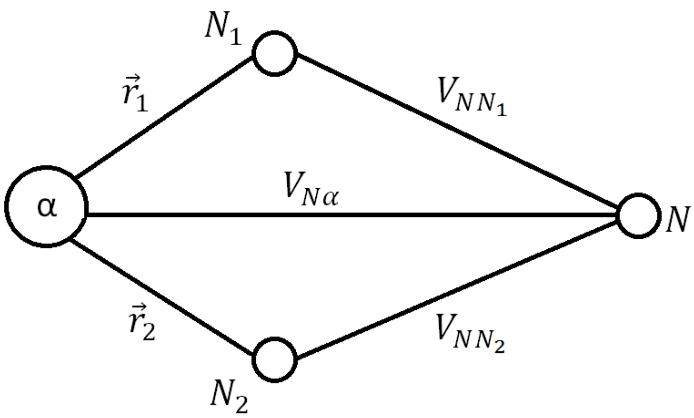

Fig. 6. The cluster form α-N-N of the systems ^6Li and ^6He.

Then the interaction of the nucleon with such a system can be divided into the interaction with the α-particle and with two external nucleons. Thus, inter-channel interactions can be written as follows:

$$V_{ij} = V_{\alpha N}(r)\delta_{ij} + \int d\xi_1 d\xi_2 \, \Psi_j(\xi,\xi_1,\xi_2) \sum_k V_{NN_k}(\vec{r}-\vec{r}_k,\vec{\sigma},\vec{\sigma}_k,\vec{\tau},\vec{\tau}_k)\Psi_i(\xi,\xi_1,\xi_2)$$

where the coordinate ξ - includes radial, spin and isospin variables.

For convenience, we will use the effective αN-interaction $V_{\alpha N}(r)$ parameterized by a Gaussian:

$$V_{\alpha N}(r) = V_0^{(\alpha)} e^{\left(-r/r_0^{(\alpha)}\right)^2},$$

where $V_0^{(\alpha)} = -47.32\ MeV$ for p-wave interaction, $r_0^{(\alpha)} = 2.30\ fm$.

The effective nucleon-nucleon interaction was chosen in the following representation:

$$V_{NN_k}(\vec{r}-\vec{r}_k,\vec{\sigma},\vec{\sigma}_k,\vec{\tau},\vec{\tau}_k) = V_c(|\vec{r}-\vec{r}_k|)[W + BP_\sigma - HP_\tau - MP_\sigma P_\tau],$$

где $P_\sigma = \frac{1}{2}(1+\vec{\sigma}\vec{\sigma}_k)$, $P_\tau = \frac{1}{2}(1+\vec{\tau}\vec{\tau}_k)$, $V_0(|\vec{r}-\vec{r}_k|) = V_0 e^{-(|\vec{r}-\vec{r}_k|\mu)^2}$

Or in another, more convenient way:

$$V_{NN_k}(\vec{r}-\vec{r}_k,\vec{\sigma},\vec{\sigma}_k,\vec{\tau},\vec{\tau}_k) = V_c(|\vec{r}-\vec{r}_k|)[a_0 + a_\sigma(\vec{\sigma}\vec{\sigma}_k) + a_\tau(\vec{\tau}\vec{\tau}_k) + a_{\sigma\tau}(\vec{\sigma}\vec{\sigma}_k)(\vec{\tau}\vec{\tau}_k)]$$

In the literature [2-5] there are many sets of parameters for this interaction. In this paper, we use two different and most modern sets of parameters:

Table 1. The parameters of various two-nucleon potentials used in this report.

Name	Range $\mu\ (fm^{-2})$	Strength $V_0\ (MeV)$	a_0	a_σ	a_τ	$a_{\sigma\tau}$
Hasegawa #2	0.16	-6	0.0313	0.0104	-0.0313	-0.2882
	1.127	-546	0.3569	-0.0382	-0.1234	-0.1037
	3.4	1655	0.3722	0.0489	-0.1259	-0.0996
TT	0.46	-72.98	-0.9608	-0.0868	-0.1793	-0.0241

We assume that nucleons in ^6He and ^6Li are occupying only $p_{3/2}$ shells and use oscillator radial wave functions of the p-shell nucleons N_1 and N_2,:

$$\varphi_k(r) = \sqrt{\frac{8}{3\sqrt{\pi}}}\frac{r}{r_0^{5/2}} e^{-\frac{r^2}{2r_0^2}}$$

where $r_0 = 2.73$ for nucleons in the ^6He and $r_0 = 2.43$ for nucleons in the ^6Li.

We decompose the central interaction into multipoles:

$$V_c(|\vec{r}-\vec{r}_k|) = \sum_l V_l(r,r_k)\frac{4\pi}{2l+1}\left(Y_l(\theta,\varphi)\cdot Y_l(\theta_k,\varphi_k)\right)$$

The expansion coefficients are found by the following formula:

$$\mathrm{V_l(r, r_k)} = \frac{2l+1}{2}\int_{-1}^{1}\mathrm{V_0}e^{-(|\vec{\mathrm{r}}-\vec{\mathrm{r}}_\mathrm{k}|\mu)^2}\mathrm{P_l(cos\theta)dcos\theta}$$

Thus, for transition potentials, we have:

$$V_{ij} = V_0^{(\alpha)}e^{\left(-r/r_0^{(\alpha)}\right)^2} + 2\pi\int dr_1\varphi_{j(r_1)}\varphi_{i(r_1)}r_1^2\int_{-1}^{1}\mathrm{V_0}e^{-(|\vec{\mathrm{r}}-\vec{\mathrm{r}}_1|\mu)^2}\mathrm{P_l(cos\theta)dcos\theta}\int dr_2 dr_2\varphi_{j(r_2)}\varphi_{i(r_2)}r_2^2 B_l^{(1)} + 2\pi\int dr_2\varphi_{j(r_2)}\varphi_{i(r_2)}r_2^2\int_{-1}^{1}\mathrm{V_0}e^{-(|\vec{\mathrm{r}}-\vec{\mathrm{r}}_2|\mu)^2}\mathrm{P_l(cos\theta)dcos\theta}\int dr_1 dr_1\varphi_{j(r_1)}\varphi_{i(r_1)}r_1^2 B_l^{(2)}$$

where

$$B_l^{(k)} = \left\langle \left[[j_1\otimes j_2]_{j_0}\otimes j\right]_J \middle| \left(\mathrm{Y_l(\hat{r})\cdot Y_l(\hat{r}_k)}\right)\left(\mathrm{a_0 + a_\sigma(\vec{\sigma}\vec{\sigma}_k) + a_\tau(\vec{\tau}\vec{\tau}_k) + a_{\sigma\tau}(\vec{\sigma}\vec{\sigma}_k)(\vec{\tau}\vec{\tau}_k)}\right) \middle| \left[[j_1'\otimes j_2']_{j_0'}\otimes j'\right]_{J'} \right\rangle$$

– angular matrix element; j_1, j_2, j_1', j_2' – angular momenta of nucleons $\mathrm{N_1}$ and $\mathrm{N_2}$ in the initial and final state, respectively; j and j' – angular momenta of the external nucleon N in the initial and final state, respectively.

We consider the case, when the seven-nucleon system has a total angular momentum $J^\pi = 3/2^-$. Then we have the following values of the corresponding parts of the angular matrix element for $l = 0$:

Table 2. Values of the corresponding parts of the angular matrix element for $l = 0$.

Channel	11	12	13	14	15	22	23	24	25	33	34	35	44	45	55
$\hat{V}_c$	0.159	0.000	0.000	0.000	0.000	0.159	0.000	0.000	0.000	0.159	0.000	0.000	0.159	0.000	0.159
$\hat{V}_\sigma$	0.020	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.061	0.000	0.061
$\hat{V}_\tau$	0.000	0.000	0.000	0.000	0.000	-0.265	0.075	0.000	0.000	-0.212	0.000	0.000	-0.265	0.075	-0.212
$\hat{V}_{\sigma\tau}$	0.000	0.125	-0.088	0.089	-0.063	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.102	0.029	-0.082

and for $l = 2$:

Table 3. Values of the corresponding parts of the angular matrix element for $l = 2$.

Channel	11	12	13	14	15	22	23	24	25	33	34	35	44	45	55
$\hat{V}_c$	0.051	0.000	0.000	0.000	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.036	0.000	0.000	0.000
$\hat{V}_\sigma$	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.071	0.000	0.071
$\hat{V}_\tau$	0.000	0.000	0.000	0.070	-0.049	0.000	0.000	-0.119	0.034	0.000	0.034	0.000	0.000	0.000	0.000
$\hat{V}_{\sigma\tau}$	0.000	0.004	-0.003	0.041	-0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.119	0.034	-0.095

Then, using the effective nucleon-nucleon potential "Hasegawa # 2" for inter-channel potentials V_{ij} we have:

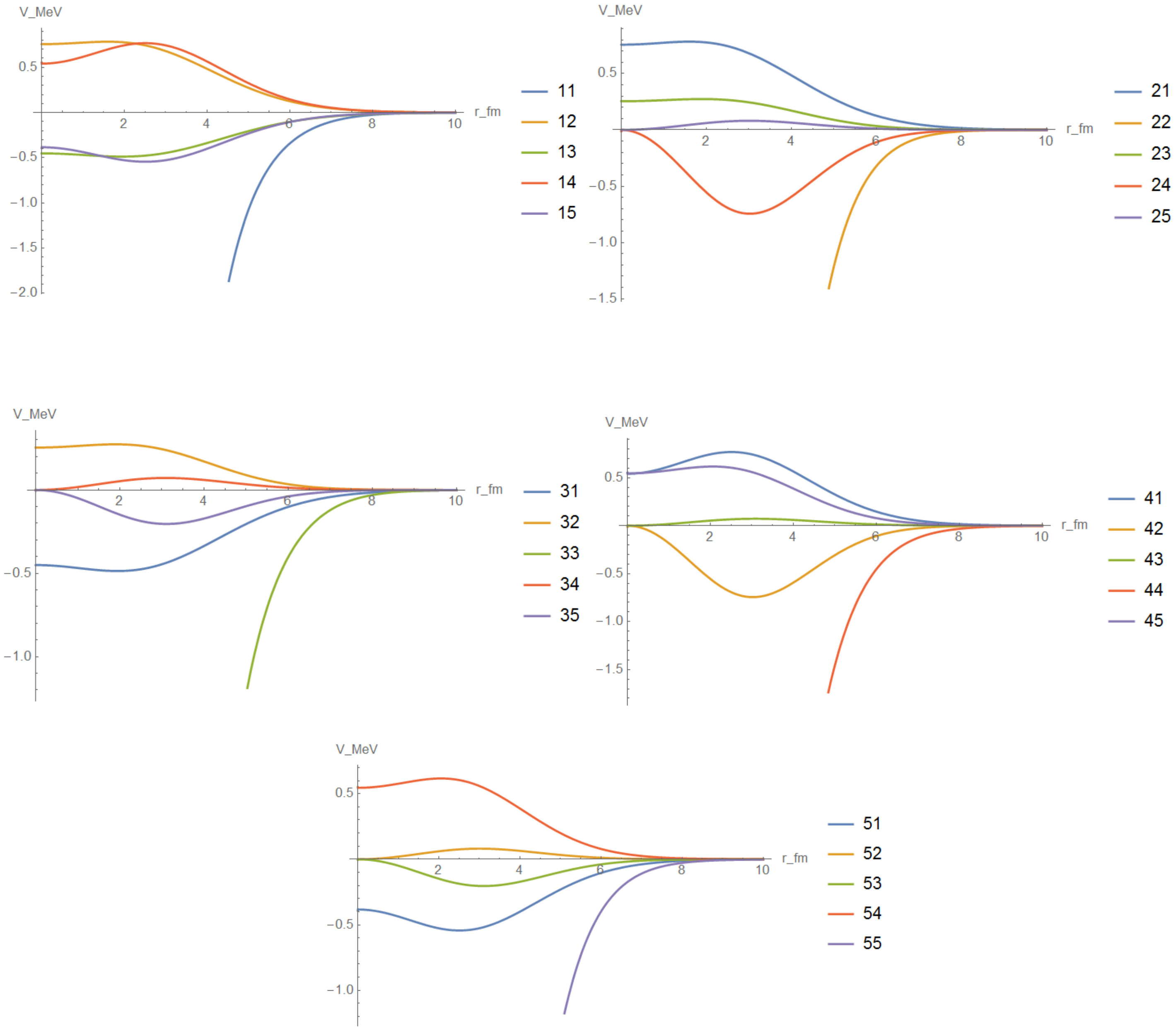

Fig. 7. Inter-channel potentials V_{ij} with using effective NN-potential "Hasegawa # 2".

And using the effective nucleon-nucleon potential “TT”:

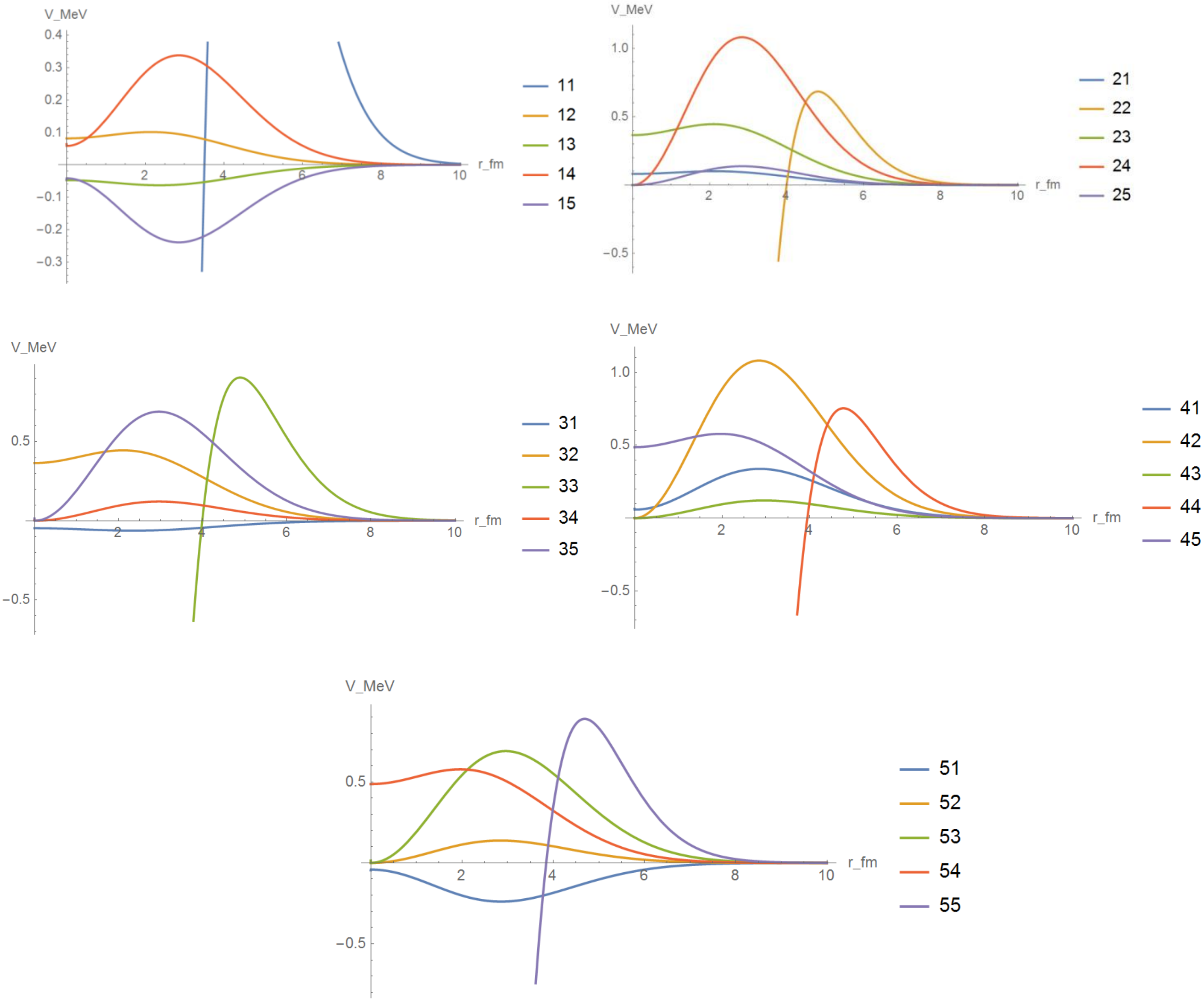

Fig. 8. Inter-channel potentials V_{ij} with using effective NN-potential “TT”.

Conclusion

We studied the scattering of protons on the ^{8}He in a two-channel approach and examined the boundary cases when the interactions $V_{3/2}$ и $V_{5/2}$ with pure isospin are either equal to each other, or each of them is equal to zero in separately.

To study proton scattering on the nucleus ^{6}He we use a more advanced, five-channel approach. We found the angular matrix elements for each of the parts of the phenomenological nucleon-nucleon potential, and also calculated the diagonal and non-diagonal inter-channel interactions V_{ij}.

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