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Azimuth distribution of particles study using Beam-Monitoring Detector for MPD (Multi-Purpose Detector)

Supervisor: Oleg Rogachevsky

Student: Alejandro Guirado García

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AZIMUTH DISTRIBUTION OF PARTICLES STUDY USING BEAM-MONITORING DETECTOR FOR MPD (MULTI-PURPOSE DETECTOR)

Alejandro Guirado García*

ABSTRACT

The distribution in the overlap region of the colliding nuclei is affected by the pressure gradients created in the initial moment of collision, which can be characterized by the coefficients of Fourier expansion of azimuth distribution in momentum coordinates. The event plane angle can be estimated via the azimuthal distributions of particles produced in the collision. We report the event-plane angle determined by the standard method, resolution of BMD detector, the directed and elliptic flow for 5000 events of Au+Au min-bias at energies of 4, 9 and 11GeV in c.m using URQMD as event generator. .

Introduction

One of the main goals in studying collision of relativistic heavy ion is to reconstruct and relate the initial geometry of collision with the observable measured by detectors. The initial geometry of a heavy-ion collision can be described in terms of the impact parameter vector connecting the centers of the colliding nuclei. The impact parameter magnitude is correlated with the size of the overlap region which is called centrality. In URQMD the impact parameter is added on x direction, thus we can define a plane with the impact parameter vector and the beam direction z, this plane is usually called reaction plane.

Section 1 shows the analysis of the azimuthal anisotropy resulting from non-central nuclear collisions using the Fourier expansion of azimuthal distributions. Section 2 discusses the results of the event plane angle obtained by using the event plane angle standard method. Section 3 show the results of the coefficients of the Fourier expansion, specifically the first two, usually called directed and elliptic flow respectively. Finally in section 4, the resolution of the BMD detector is discussed.

1 Azimuth distribution of particles

The Physics produced in Relativistic Heavy Ion Collisions(RHIC) can be described in terms of initial collision's geometry, which can be described in terms of the impact parameter vector connecting the centers of the colliding nuclei. The impact parameter magnitude can not be measured directly, one has to estimate it from the number of produced particles or the energy of spectators using experimental data, however in simulation analysis is commonly used the impact parameter(b) vector to describe it since we know it by the Monte Carlo. The impact parameter magnitude is correlated with the size of the overlap region which is called centrality. In URQMD the impact parameter is added on x direction, thus we can define a plane with the impact parameter vector and the beam direction z, this plane is usually called reaction plane.

*Department of Physics. University of Sonora, México.

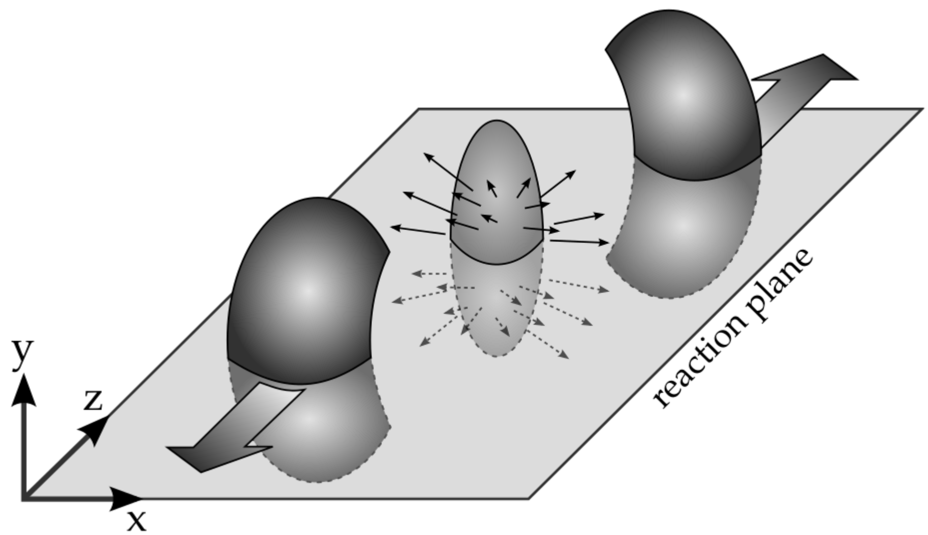

Figure 1: Illustration of a non-central heavy ion collision with an elliptical reaction volume which is symmetric with respect to the reaction plane.

In non-central collisions the shape of the interaction region depends on the impact parameter of the collision. If the impact parameter is zero, the shape of the interaction region is a sphere centered at the origin. But, if the impact parameter is not null, the interaction shape could be an ellipse creating an azimuthal transverse momentum distribution due the anisotropy of the reaction volume. To characterize this phenomenon, it is used a Fourier expansion of the moment distribution with respect to the reaction plane. For example defining a quantity $r(\phi)$ which can be dP_T/dϕ where dP_T is the total transverse momentum of particles emitted at azimuthal angle ϕ. Experimentally this function can be constructed by data, regarding to periodicity of the function $r(\phi)$ is possible to write it in the form of Fourier expansion.

$$r(\phi) = \frac{x_0}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty}[x_n cos(n\phi) + y_n sin(n\phi)] \tag{1}$$

As we know for Fourier analysis the coefficients in the Fourier expansion are integrals with weight proportional to cosines and sines. How ever, in this case we are treating with a finite number of particles, so the integral becomes a simple sum over all particles.

$$\begin{aligned} x_n &= \sum_{\nu} r_\nu cos(n\phi_\nu) \\ y_n &= \sum_{\nu} r_\nu sin(n\phi_\nu) \end{aligned} \tag{2}$$

Here the sum runs in all particles and ϕ_ν is the azimuths angle of the ν particle. With these Fourier analysis it is possible to find important signatures of flows distribution.

With the previous analysis we can find a more general equation to describe the azimuths distribution for particles in energy, transverse momentum and rapidity region. Defining a function r(ϕ) as $E\frac{d^3N}{d^3p}$. Here, we consider the E as the energy, N a measurable quantity, p as the three momentum of each particle and the angle now is measured at other plane with angle $\phi - \Psi_{RP}$. This change of plane is very important since we are taking into account the reaction plane of the event(Ψ_{RP}) and the angle(ϕ) of each particle which lies in xy plane.

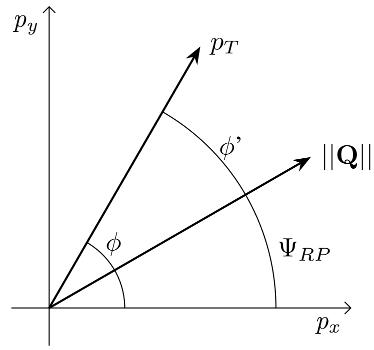

Figure 2: Azimuth angle of particles in momentum coordinates ϕ, the reaction plane angle Φ_{RP} and ϕ' is the rest of the azimuth angle of particles and the reaction plane angle and $||\mathbf{Q}||$ is the vector used in the standard event plane angle method discussed in section 2.

To make the calculus easier is common to reformulate the integral over a particle's momentum in terms of rapidities and transverse momenta. So we need to calculate the Jacobian determinant, but first the transformation equations are:

$$\begin{aligned} p_x &= p_T cos(\theta) \\ p_y &= p_T cos(\theta) \\ p_z &= \sqrt{p_T^2 + m^2} sinh(y) \end{aligned} \tag{3}$$

For simplicity we used the angle θ instead of $\phi - \Psi_{RP}$. The Jacobian is:

$$\begin{aligned} J(p_x, p_y, p_z)_{p_T,\theta,y} &= \left| \tfrac{\partial[p_x,p_y,p_z]}{\partial[p_T,\theta,y]} \right| \\ &= p_T cosh(y) \sqrt{p_T^2 + m^2} \end{aligned} \tag{4}$$

Since we are considering the particles as free particles without any specific potential acting on them, the 4-momentum vector loss one degree of freedom, we can rewrite it as $p^3 = p(p_x, p_y, p_z) = p(p_T, y)$ [7]. So the transverse mass and the energy are defined as:

$$\begin{aligned} m_T &= \sqrt{p_T^2 + m^2} \\ E &= cosh(y) m_T \end{aligned} \tag{5}$$

And the Jacobian of equation (4) reduce to:

$$J(p_x, p_y, p_z)_{p_T,\theta,y} = p_T E \tag{6}$$

Now we are able to calculate the coefficients of the Fourier expansion of the azimuth distribution (**??**)

$$\begin{aligned} x_0 &= \int_D E \frac{d^3N}{d^3p} d\theta \\ &= \int_D E \frac{d^3N}{p_T E dp_T d\theta dy} d\theta \\ &= \frac{d^2N}{p_T dp_T dy}\Big|_D \end{aligned} \tag{7}$$

The coefficient x_0 is determined by the equation (7) which is valuated in the domain D = D(p_T, y) but for the next equations we expressed without the evaluation. Now for the x_n and y_n coefficients.

$$
\begin{aligned}
x_n &= \int_D E\frac{d^3N}{d^3p}cos(n\theta)d\theta \\
&= \frac{d^2N}{p_Tdp_Tdy}\left[\frac{\int_D E\frac{d^3N}{d^3p}cos(n\theta)d\theta}{\int_D E\frac{d^3N}{p_TEdp_Td\theta dy}d\theta}\right] \\
&= \frac{d^2N}{p_Tdp_Tdy}\langle cos(n\theta)\rangle \\
y_n &= \frac{d^2N}{p_Tdp_Td\theta dy}\langle sin(n\theta)\rangle
\end{aligned} \tag{8}
$$

Recalling that the particle source is symmetric with respect to the reaction plane the y_n coefficients are zero and the average is over all the particles in the range of rapidity and transverse momentum wanted. Plugging the equations (7) and (8) in equation (1) and remembering that the angle $\theta = \phi - \Psi_{RP}$. The azimuth angle distribution is:

$$
\begin{aligned}
E\frac{d^3N}{d^3p} &= \frac{1}{2\pi}\frac{d^2N}{p_Tdp_Tdy}\left[1 + 2\sum_{n=1}^{\infty}\langle cos[n(\phi - \Psi_{RP})]\rangle cos[n(\phi - \Psi_{RP})]\right] \\
&= \frac{1}{2\pi}\frac{d^2N}{p_Tdp_Tdy}\left[1 + 2\sum_{n=1}^{\infty}v_n(p_T, y)cos[n(\phi - \Psi_{RP})]\right]
\end{aligned} \tag{9}
$$

Where E, N ,p^3 , p$_T$, ϕ and η are the particle's energy, yield, total 3-momentum, transverse momentum, azimuthal angle and rapidity, respectively. This results is very important because of the simplicity of the term v_n can be compared to theoretical predictions or to simulations for detector acceptance. In addition we can simulate till the n order of v_n coefficient, each one is related to initial conditions of collision. It is important to mention that he coefficients are usually called harmonic coefficients. We are going to study in more detail only the first two coefficients.

2 Collective flow at RHIC

The collective flows are defined as the v_n coefficients in equation (9)

$$v_n = \langle cos[n(\phi - \Psi_{RP})]\rangle \tag{10}$$

Depending of the number n it change its name according to the type of flow which are related. Using the next image to express better the idea:

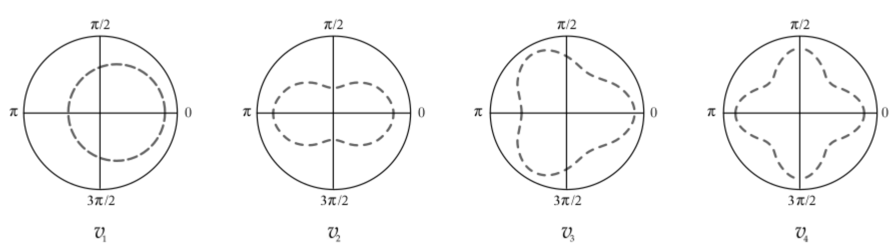

Figure 3: Geometric illustration of the first 4 v_n coefficients. This image was taken from [5].

If the reaction plane angle is null is possible to the first three coefficients v_n in terms of p_T, p_x, py, the first three coefficients are:

$$
\begin{aligned}
v_1 &= \left\langle \frac{p_x}{p_T} \right\rangle \\
v_2 &= \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle \\
v_3 &= \left\langle \frac{p_x^3 - p_x p_y^2 - p_x}{p_T^3} \right\rangle
\end{aligned}
\tag{11}
$$

The first two coefficients are specially important in the analysis of flows in non-central heavy ion collisions. The v_1 is usually called "directed flow" and represent an overall shift of the distribution in the transverse plane. On the other hand the second harmonic coefficient represent an ellipse in the transverse distribution of momentum. Following the same reasoning the third harmonic coefficient is called triangular flow and so on.

3 Event plane angle

Since in experiments it is not possible to know the initial conditions of nuclei collisions we relate the physics produced with observable such as multiplicity, transverse momentum, event angle plane, etc. Knowing the event angle plane permit us to study the collective flows mentioned above, study the resolution of a detector, among other things. The estimation of the reaction plane angle is called event plane angle, it is possible to calculate the event plane angle with experimental data or simulations. In this case since we do not have experimental data yet, we focus only at the simulation method. This method is called the standard event plane method,so it is possible to estimate Ψ_{RP}. Practically in this method we measure how the particles are distributed and its weights in the transverse plane. So, we define a two dimensional vector in transverse plane $\mathbf{Q}$

$$
\begin{aligned}
Q_{n,x} &= \sum_i^N w_i cos(n\phi_i) = \mathbf{Q}_n cos(n\Psi_n) \\
Q_{n,y} &= \sum_i^N w_i sin(n\phi_i) = \mathbf{Q}_n sin(n\Psi_n)
\end{aligned}
\tag{12}
$$

where the sum goes over all particles i used in the event plane calculation. Where ϕ, ϕ', Ψ_{RP} and w_i are the lab azimuths angle, different from azimuths angle and reaction plane and weight for particle i. Transverse momentum is a common choice as weight, since it increase linearly as the coefficients of the Fourier expansion $v_n(p_T, \eta)$, so $w = p_T$. Analyzing the Figure 1 we can find the values of the angle ϕ in terms of p_x and p_y.

$$
cos(\phi) = \frac{p_x}{p_T} \qquad sin(\phi) = \frac{p_y}{p_T} \tag{13}
$$

Since the vector Q_n in equation (12) is the same we can calculate the magnitude and divide the y component by the x component.

$$
\frac{Q_{n,y}}{Q_{n,x}} = \frac{sin(n\Psi_n)}{cos(n\Psi_n)} \tag{14}
$$

Solving for Ψ_n we obtain.

$$
\begin{aligned}
\Psi_n &= \frac{1}{n} Tan^{-1} \left[\frac{Q_{n,y}}{Q_{n,x}} \right] \\
&= \frac{1}{n} Tan^{-1} \left[\frac{\sum_i^N w^i cos(n\phi_i)}{\sum_i^N w^i sin(n\phi_i)} \right]
\end{aligned}
\tag{15}
$$

With equation (15) we can calculate the event plane for a detector if know the orientation in the transverse plane and the weight. If we use transverse momentum as weight and with the equations (13), the event plane angle is reduced to:

$$\begin{aligned}\Psi_n &= \frac{1}{n} Tan^{-1}\left[\frac{\sum_i^N p_T^i cos(n\phi_i)}{\sum_i^N p_T^i sin(n\phi_i)}\right] \\ &= \frac{1}{n}\left[\frac{\sum_i^N py^i}{\sum_i^N px^i}\right]\end{aligned} \tag{16}$$

Equation (16) is easy to evaluate since we know the momentum coordinates for each particle in all the events. This method is made event by event, so each event has its own event plane angle determined by this method. However we calculate this angle using the particles which reach the detector, since we know that the multiplicity is related with the impact parameter is common to adjust the coefficients with a parameter defined by each detector called resolution.

For detectors with high granularity as BMD detector we can use the energy deposited by particle or multiplicity per cell as weight and the angle ϕ is the ith-cell's azimuthal angle measured from the center of the hodoscope to the cell centroid.

$$\Psi_n = \frac{1}{n}\left[\frac{\sum_i^N E_i y_i}{\sum_i^N E_i x_i}\right] \tag{17}$$

The event angle plane was calculated for BMD detector was calculated using equations (16) and (17).

4 Event plane angle resolution

In practice the reaction plane is not possible to measure it, instead is common used the event plane which can be obtained by different experimental methods such as "correlation between flow angles of independent sets of particles" [1], but in simulation we are able to use the reaction plane and event plane angle. To find the event plane resolution we need to compare the event plane angle (15) with the true reaction plane angle given by Monte Carlo simulation for the n harmonic in a narrow centrality bin. The event plane resolution is:

$$\mathcal{R}_n = \langle cos[n(\Psi_n - \Psi_{RP})]\rangle \tag{18}$$

Where the reaction plane is given by Monte Carlo and the event plane angle using equation (15). This correction apply for the collective flow averaged in wide centrality bins. Thus, the collective flow increase since the resolution is always less than 1, unless we are using a perfect detector which is actually not yet created. The coefficients with the correction are:

$$v_n = \frac{v_{obs}}{\mathcal{R}} \tag{19}$$

Where v_n are the corrected collective flows coefficients in a narrow centrality bin. Here, some authors have discussion about how to implement this correction [6], nevertheless since we could not find the resolution of the detector well, we could not implement this correction in the code, for that reason all the flows presented are the observables flows.

5 Simulation and results

5.1 Collective flow at RHIC

For studying collective flow we can start by analyzing the coefficients v_n directly from simulation using the MCTRACKS branch in the tree created after detector's simulation using MpdROOT framework. For the calculus we used the standard event plane angle method with transverse momentum as weight to calculate the reaction plane using equation (16) selecting only the primary tracks and compared with the collective flow calculated by using the particles which reach the detector to calculate the event plane angle obtained by the standard method using weight as transverse momentum. Finally, we compared the directed and elliptic flow for pions and protons calculated by using the Monte Carlo reaction plane, for URQMD is 0 by default. We present the results according of its collision c.m energy.

- $\sqrt{S_{NN}} = 4GeV$

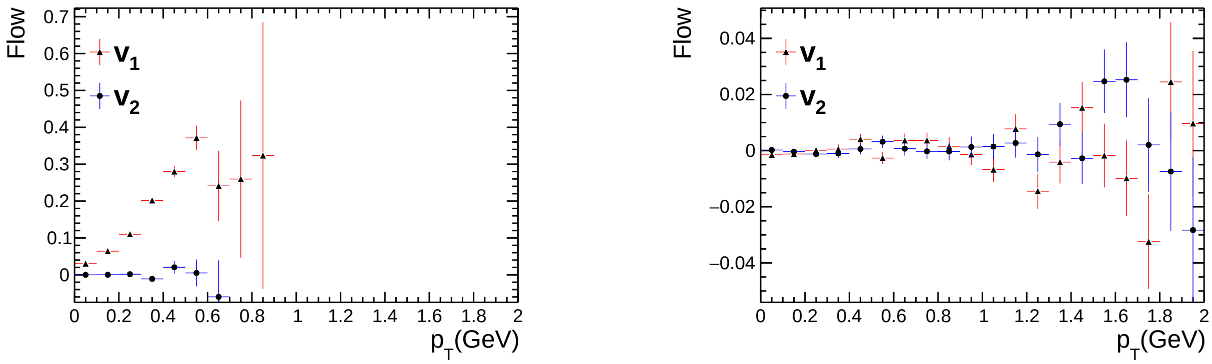

Figure 4: Directed and elliptic flow as a function of pseudorapidity and transverse momentum from minimum bias 4 GeV Au+Au using URQMD as event generator at time t=100fm. The left figure is for BMD detector points and the right figure for primary tracks.

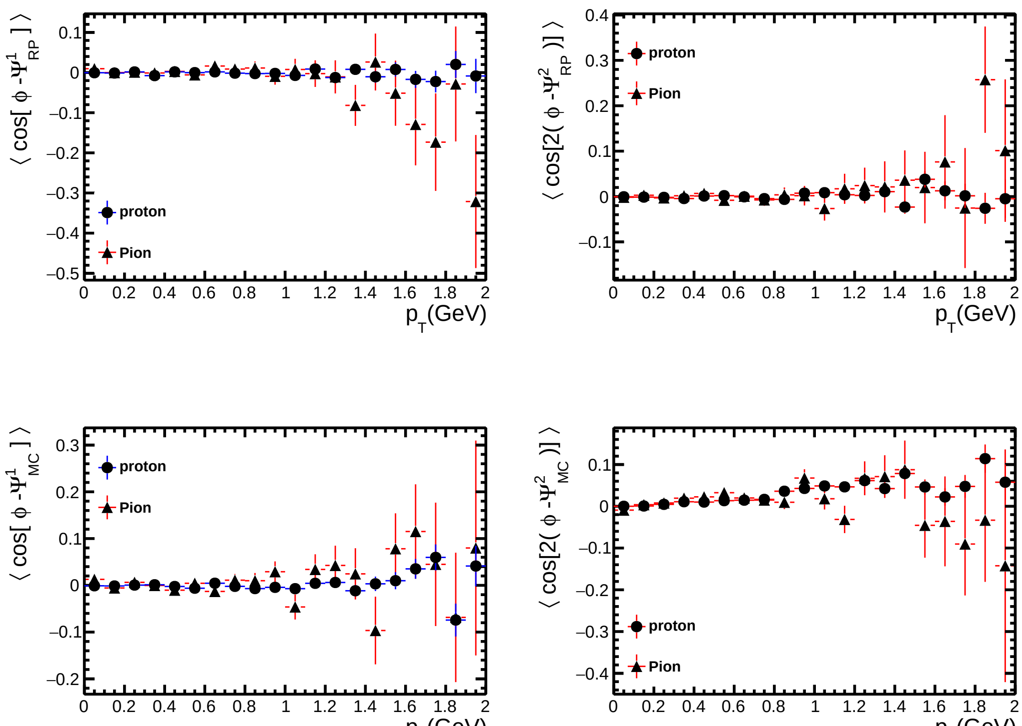

Figure 5: Directed and elliptic flow as a function of pseudorapidity and transverse momentum from minimum bias 4 GeV Au+Au using URQMD as event generator at time t=100fm calculated with primary tracks and true reaction plane angle for pions and protons.

- $\sqrt{S_{NN}} = 9GeV$

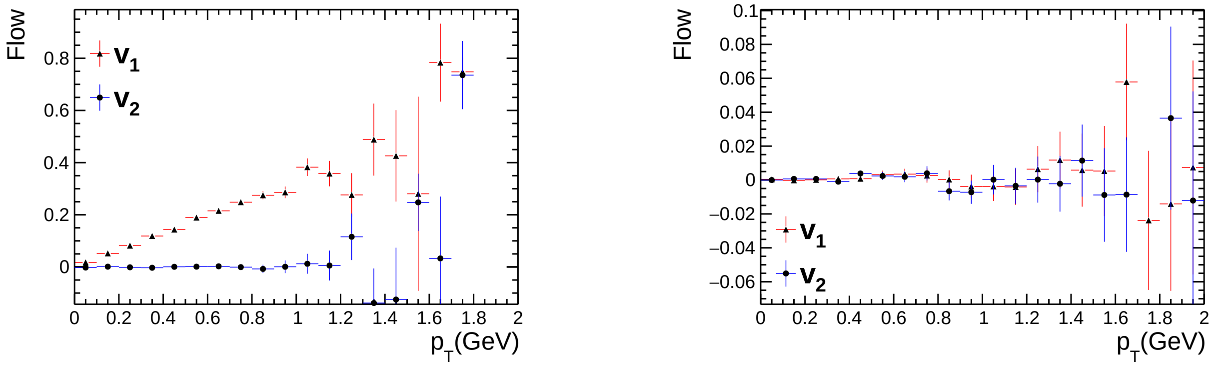

Figure 6: Directed and elliptic flow as a function of pseudorapidity and transverse momentum from minimum bias 9 GeV Au+Au using URQMD as event generator at time t=100fm. The left figure is for BMD detector points and the right figure for primary tracks.

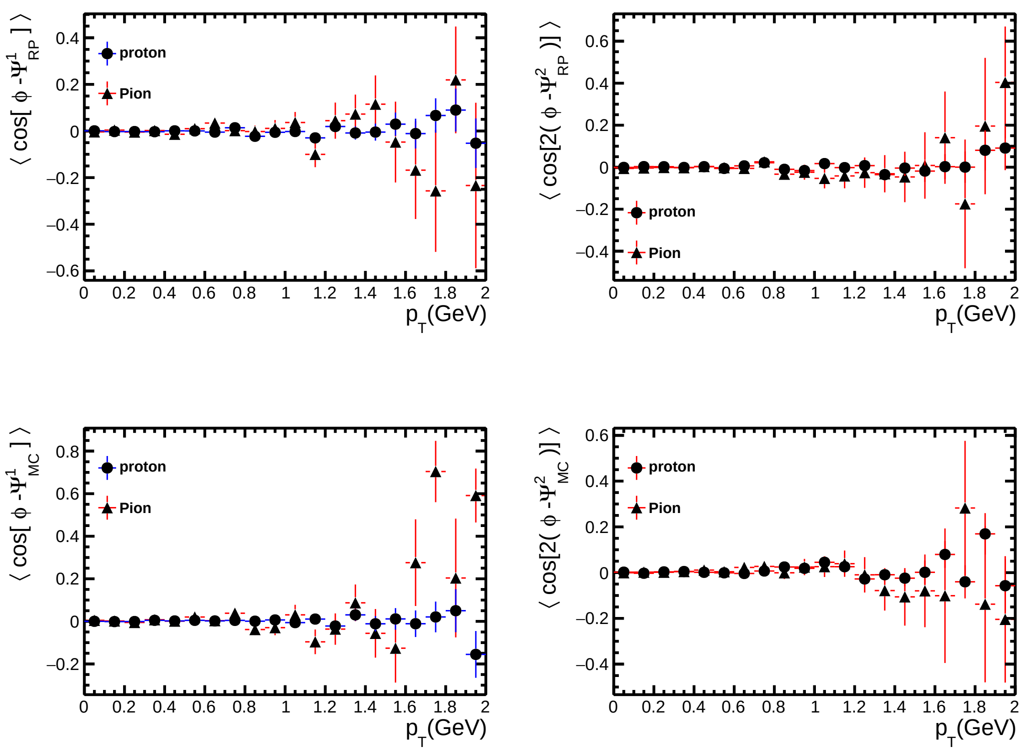

Figure 7: Directed and elliptic flow as a function of pseudorapidity and transverse momentum from minimum bias 9 GeV Au+Au using URQMD as event generator at time t=100fm calculated with primary tracks and true reaction plane angle for pions and protons.

- $\sqrt{S_{NN}} = 11GeV$

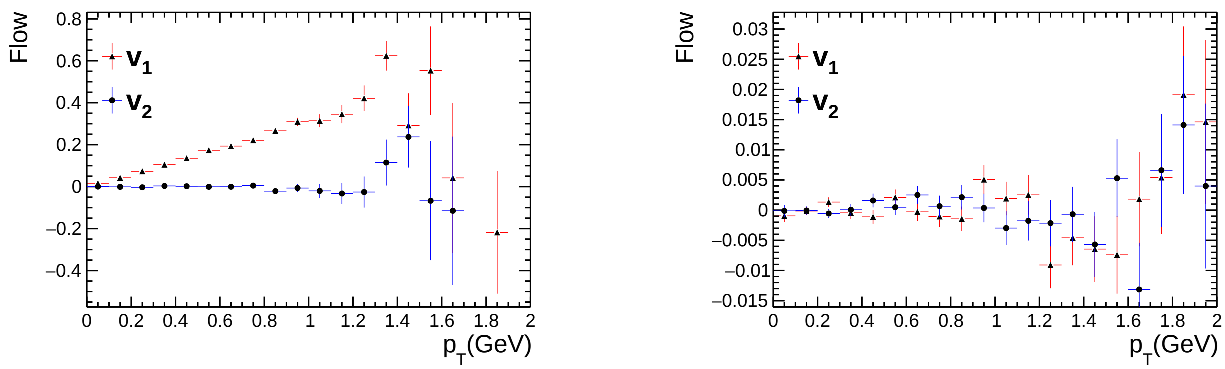

Figure 8: Directed and elliptic flow as a function of pseudorapidity and transverse momentum from minimum bias 11 GeV Au+Au using URQMD as event generator at time t=100fm. The left figure is for BMD detector points and the right figure for primary tracks.

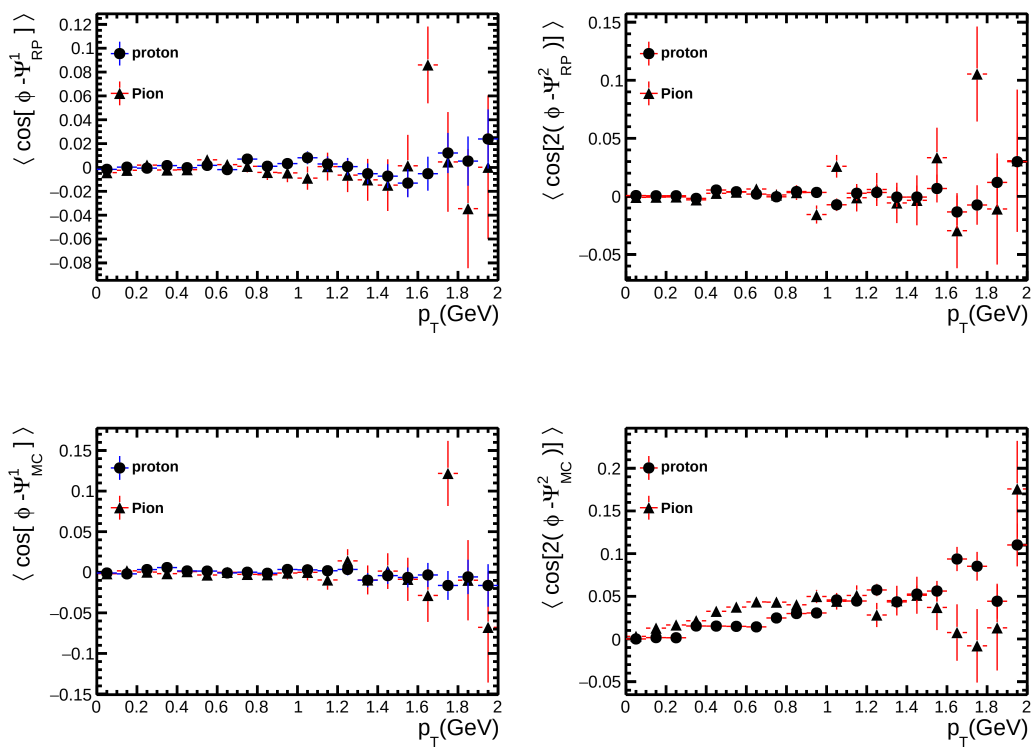

Figure 9: Directed and elliptic flow as a function of pseudorapidity and transverse momentum from minimum bias 4 GeV Au+Au using URQMD as event generator at time t=100fm calculated with primary tracks and true reaction plane angle for pions and protons.

5.2 Event plane angle

The event plane angle is calculated using the standard method which is described in section 3, at the final of this report is the code which was implemented for this method using transverse momentum for all particles in an specific range of momentum and pseudorapidity. In this analysis the range are for $p_t < 2GeV$ and $1.9 < |\eta| < 3.9$. Then, the event plane angle method is calculated measuring the energy loss and multiplicity per cell using equation (17).

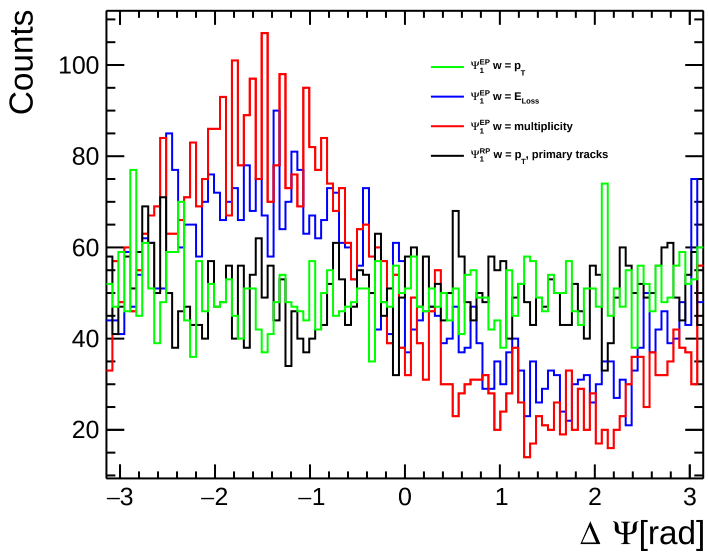

Figure 10: Difference from true event plane angle and event plane angle for the first harmonic calculated using the standard method for different weights, the simulation minimum bias 4 GeV Au+Au using URQMD as event generator at time t=100fm.

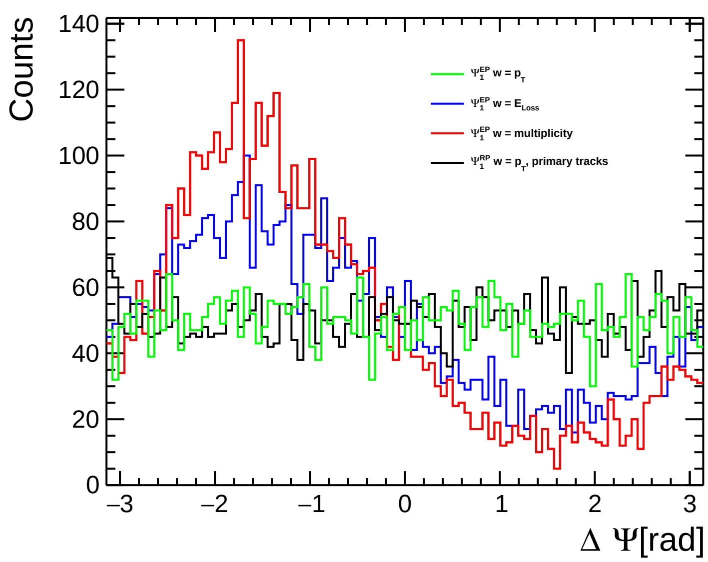

Figure 11: Difference from true event plane angle and event plane angle for the first harmonic calculated using the standard method for different weights, the simulation minimum bias 9 GeV Au+Au using URQMD as event generator at time t=100fm.

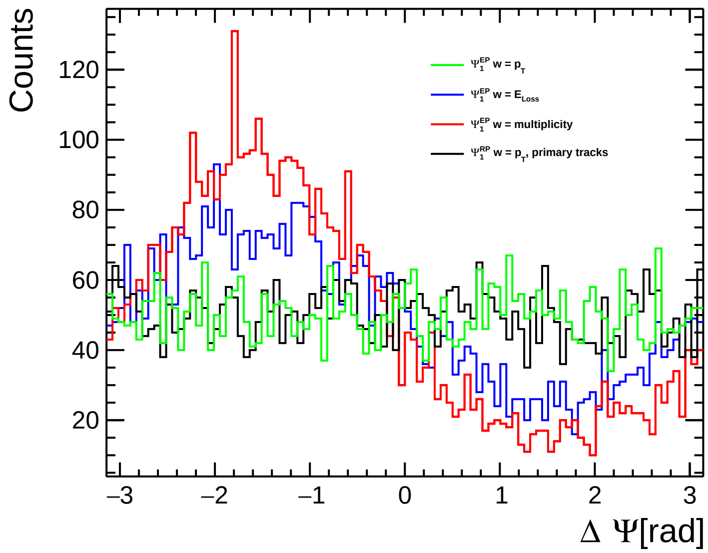

Figure 12: Difference from true event plane angle and event plane angle for the first harmonic calculated using the standard method for different weights, the simulation minimum bias 11 GeV Au+Au using URQMD as event generator at time t=100fm.

5.3 Event plane angle resolution

As we can see in the figures 10-12 the difference from the true reaction plane angle is between 1 and 2 rad which clearly does not match. So if we calculate the resolution with equation (18), remembering that for URQMD the reaction plane angle is 0, so $\Psi_{RP} = 0$, letting the resolution of BMD detector for the first harmonic:

$$R_1 = \langle cos[\Psi_1] \rangle \tag{20}$$

Here the average is over events in a centrality bin, some comments about centrality can be found in the first appendix. We calculated the resolution for the BMD detector using as weight the transverse momentum, multiplicity and energy loss per cell.

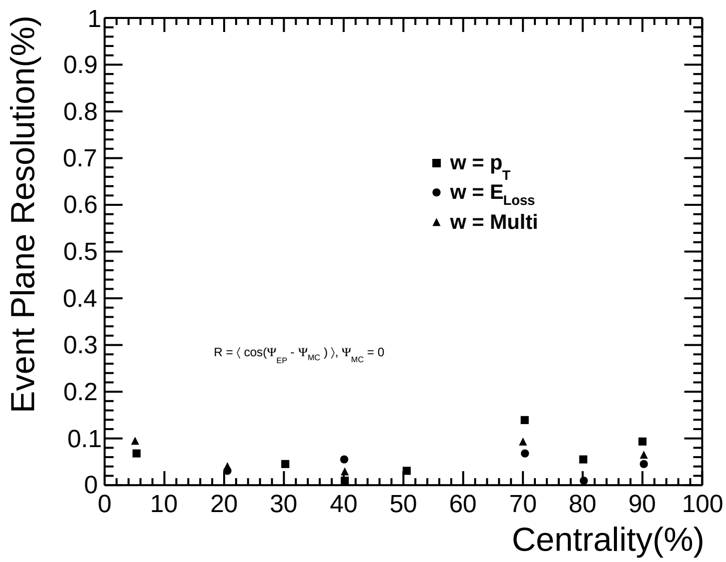

Figure 13: Resolution for BMD detector, the event plane angles were calculated by using the standard method with different weights at function of centrality. The simulation minimum bias 11 GeV Au+Au using URQMD as event generator at time t=100fm.

The resolution is not well described because the determination of event plane angle is wrong, we can compare the results with similar detectors [4], [6] and [10].

Summary

A macro for calculate the flows, event plane angle with different weights and resolution were made. The method can be found in v2std.C macro in mpdroot directory. The reasons why the results does not match with other results specially for resolution are discussed however till today these are the latest results. Of course there is a lot of work to finish. This summer were very useful for me to develop my knowledge in mpdroot simulations, macros for physical analysis and how to work with root.

Acknowledgment

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Apendix

Brief comments about centrality

If you want a more detailed explanation of centrality determination in relativistic heavy ion collisions you can read " *Geometric relation between centrality and the impact parameter in relativistic heavy-ion collisions*" [8] Is very useful to find a relation between centrality and impact parameter for simulations because we are able to control it and analyze the results in term of other parameters such as multiplicity of produced particles, number of participants, etc. To all this parameters we will call them n to avoid being repetitive. The centrality C is usually defined as:

$$C(N) = \sum_{n=N}^{\infty} P(n) \tag{21}$$

Where $P(n)$ is the probability to find the value n and $C(N)$ is the probability of obtain an event n with n larger or equal to N($n \geq N$). Now we need to identify how to calculate the $P(n)$. This probability needs to be related with the n, but if we want to relate the variable n with the impact parameter we need to calculate the probability of obtain a n value with an impact parameter given. So the probability is:

$$P(n) = \int_0^{\infty} \frac{2\pi b db P(b) P(n|b)}{\sigma_{inel}} \tag{22}$$

The diferential area of collision is of course related with b and it is $dA = 2\pi b db$, $P(b)$ is the probability of an inelastic collision event at impact parameter b and $P(n|b)$ is the probability of getting a n value given an impact parameter b. Then, the centrality es:

$$C(N) = \sum_{n=N}^{\infty} \int \frac{2\pi b db P(b) P(n|b)}{\sigma_{inel}} \tag{23}$$

The equation (23) is properly normalized just recalling the definition of inelastic cross section and that the sum over all probabilities must be one. Here we are going to make 3 assumptions to continue in an easy but useful way.

- For heavy ion nuclei we can consider the continuous limit of n.

$$\sum_{n=N}^{\infty} \approx \int_{N}^{\infty} dn = \int_{0}^{\infty} dn\theta(n-N) \tag{24}$$

The $\theta(n-N)$ function in the last equation is just a step function which returns value 1 if n≥N and 0 otherwise.

- Considering a large set of values of n, we can estimate the behavior of n around the mean value of the set ($\bar{n}(b)$). So in a very rough approximation:

$$P(n|b) = \delta(n - \bar{n}(b)) \tag{25}$$

- While the impact parameter increases the n value decreases till two nuclei do not collide. This means that $\bar{n}(b)$ is a monotonically decreasing function of b. Substituting $\bar{n}(b)$ with $b(N)$ since we can expect that b(N) is a monotonically decreasing function as well.

Using the 3 assumptions commented before we get:

$$C(N) = \int_{0}^{\infty} \frac{2\pi b db P(b)}{\sigma_{inel}} \theta(b(N) - b) \tag{26}$$

To integrate the equation (26) we just need to use the step function to change the limits of integration. So if b≤ $b(N)$ the step function is 1 otherwise is 0. So the upper limit of integration is chaged to $b(N)$, getting finally

$$C(N) = \frac{1}{\sigma_{inel}} \int_{0}^{b(N)} 2\pi b db P(b) = \frac{\sigma_{inel}(b(N))}{\sigma_{inel}} \approx \frac{\pi b(N)^2}{\sigma_{inel}} \tag{27}$$

The last approximation is very useful at centrality determination since it is easier to know the value of impact parameter for a given N value than obtain the inelastic cross section obtained by a given parameter b. With equation (27) we can compute the value of centrality in terms of impact parameter, the value of inelastic cross section is taken from [9] getting $\sigma_{inel} = 7.05b$ where $b = 10^{-28}m^2$. So for our purpose we need centrality values of 5%, 20%, 30%, 40%, 50%, 60%, 70%, 80% and 90%. Solving for impact parameter we have:

$$b = \sqrt{\frac{C\sigma_{inel}}{\pi}} \tag{28}$$

Computing for impact parameters for each centrality value:

C(b)	5%	20%	30%	40%	50%	60%	70%	80%	90%
b	3.349	6.699	8.205	9.474	10.592	11.603	12.533	13.398	14.211

Table 1: Computed values of impact parameter using the equation (27) as a function of centrality.

Standard Method Event Plane Angle Code

```
//Open the tree
//Define Pointers to Data
//Iterate over events
//------------------------STANDARD ANGLE METHOD--------------------------------
        Double_t Qcos=0., Qsin=0.;
        Double_t AELoss[162] = {0};
```

```
  Double_t QcosELOSS=0., QsinELOSS=0.;
  Double_t AEMulti[162] = {0};
  Double_t QcosMULTI=0., QsinMULTI=0.;

     for (Int_t iPoint = 0; iPoint < bmdPoints->GetEntriesFast(); iPoint++)
     {
             BmdPoint* bmdPoint = (BmdPoint*) bmdPoints->UncheckedAt(iPoint);
     Double_t z = bmdPoint->GetZ();
     Double_t px = bmdPoint->GetPx();
     Double_t py = bmdPoint->GetPy();
     Double_t pz = bmdPoint->GetPz();
     Double_t pt = TMath::Sqrt(px*px + py*py);
     Double_t phi = TMath::ATan2(py,px);
     Int_t NumberRing = bmdPoint -> GetDetectorID();
             Int_t  NumberCell = bmdPoint->GetCellID();
     Double_t Eloss = bmdPoint -> GeteLoss();
     Double_t eta = 0;
     Double_t P = TMath::Sqrt( px*px + py*py + pz*pz);
     if(TMath::Abs(P- pz)>0.0000000001)
         {
              eta = 0.5*TMath::Log((P+pz)/(P-pz));
     }
     //------------------------------Event Plane Angle Method------------
//------------------------pT------------------------------------------
     if( 1.9<=TMath::Abs(eta) &&   TMath::Abs(eta) <3.9  ){
                 Qcos += pt*TMath::Cos(phi);
                 Qsin += pt*TMath::Sin(phi);
     }
//-------------------------------------------------------------------
//----------------Energy Loss & Multiplicity per cell------------
 if( NumberRing ==1 ){
   for (int i = 0; i<12;i++ ){
     if ( NumberCell == i ){
       AELoss[i] += Eloss;
       AEMulti[i] ++;
     }
   }
 }
   if( NumberRing ==2 ){
     for (int i = 0; i<18;i++ ){
       if ( NumberCell == i ){
         AELoss[i+12] += Eloss;
         AEMulti[i+12] ++;
       }
     }
   }
     if( NumberRing ==3 ){
       for (int i = 0; i<24;i++ ){
         if ( NumberCell == i ){
           AELoss[i+30] += Eloss;
           AEMulti[i+30] ++;
         }
       }
     }
       if( NumberRing ==4 ){
         for (int i = 0; i<30;i++ ){
           if ( NumberCell == i ){
             AELoss[i+54] += Eloss;
             AEMulti[i+54] ++;
```

```
            }
          }
        }
        if( NumberRing ==5 ){
          for (int i = 0; i<36;i++ ){
            if ( NumberCell == i ){
              AELoss[i+84] += Eloss;
              AEMulti[i+84] ++;
            }
          }
        }
        if( NumberRing ==6 ){
          for (int i = 0; i<42;i++ ){
            if ( NumberCell == i ){
              AELoss[i+120] += Eloss;
              AEMulti[i+120] ++;
            }
          }
        }//End BMD loop
//-----------------Event Plane Angles--------------------------------------
 Double_t PhiEP_PT = (1/Num_Harm)*TMath::ATan2(Qsin,Qcos);
 Double_t PhiEP_ELOSS = (1/Num_Harm)*TMath::ATan2(QsinELOSS,QcosELOSS);
 Double_t PhiEP_MULTI = (1/Num_Harm)*TMath::ATan2(QsinMULTI,QcosMULTI);
 }//End event loop
```