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**FINAL REPORT ON THE  
START PROGRAMME**

Comparison of experimental data and theoretical calculations of cross-sections of neutrino-nucleon interactions of GEV energies by the chi-square method using the example of experimental data NOvA and the “running axial mass” model

**Supervisor:**

Samoylov O.B.

**Student:**

Tumentseva A.A., Russia  
Moscow State University

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# 1 Abstract

This report examines: pressing issues in neutrino physics, a model featuring a running axial mass, the NOvA experiment, and a comparison of the theoretical model with experimental data.

The objective of the study was to assess the model with a running axial mass in quasi-elastic scattering and to contrast theoretical projections with data from the NOvA near detector. This detector offers a unique chance to thoroughly investigate neutrino-nucleus interaction cross-sections across the energy spectrum from hundreds of MeV to several GeV.

## 2 Introduction

### 2.1 Neutrino

Research in the field of neutrino physics occupies one of the leading places in modern science, as it opens up new perspectives for understanding the fundamental laws of nature. Neutrinos are one of the most common elementary particles in the universe. Neutrinos come from a variety of cosmic sources, including stars, quasars, supernovae, active galactic nuclei, pulsars, and there are also so-called relic neutrinos that appeared during the formation of galaxies. The key feature of neutrinos is their negligible mass and extremely weak interaction with matter. Due to this, they are able to travel vast distances with virtually no loss of energy, which makes them extremely valuable for fundamental research. Analysing the behaviour of neutrinos can allow scientists to obtain information about the most remote regions of our galaxy and reconstruct the processes that took place in the early universe. At the same time, the weak interaction of neutrinos with matter particles makes their detection extremely difficult [1].

Currently, the problem of neutrino interaction with matter is becoming particularly relevant. Modern experiments such as NOvA [2], JUNO, DUNE, Hyper-Kamiokande require a long-term set of statistics and complex models for data interpretation. A deeper understanding of this process is necessary to develop new methods for detecting neutrinos, and also allows us to set and solve problems in the search for new physical phenomena beyond the Standard Model. Modern experiments seek not only to improve the sensitivity of detectors but also to find out whether neutrinos have unusual properties, such as sterile states or a non-zero magnetic moment, which can significantly expand our understanding of the structure of matter. The detailed measurement of the double differential cross section allows testing nuclear models and correcting event generators, as well as distinguishing between different channels of interaction and understanding where theoretical models differ from experiments.

Thus, the study of neutrino interactions with matter remains a key area combining basic science, technological innovation, and the search for answers about the origin of the universe.

### 2.2 Interaction

Neutrinos interact with matter in several basic ways, and each type of interaction has its own effect on the cross section, measured by the energies and angles of escape of leptons in the final state.

The main types of neutrino interactions with matter:

- Quasi-elastic (QE) scattering. The neutrino interacts with a single nucleon inside the nucleus, knocking it out and creating a lepton. For example,  $\nu_\mu + n \rightarrow \mu^- + p$ . This is the main type of event at neutrino energies of about 1-2 GeV, which prevail in NOvA detectors. Such processes produce a relatively simple final topology of events, often with two particles the noticeable lepton and hadron.
- Resonant (RES) interaction. The transfer of energy from neutrinos excites the nucleon to a state of resonance (for example, delta resonance), which then decays to form one or more pions along with a lepton. The mechanism is more complex than that of QE, and appears at slightly higher energies.
- Deep inelastic scattering (DIS). At even higher energies, the neutrino destroys the nucleon, forming

multiple fragments — shower of hadrons (primary particles).

- Interaction with meson exchange (MEC, also 2p2h). These are processes in which a neutrino interacts with two strongly correlated nucleons in the nucleus at once, knocking them out of the nucleus at the same time. The data is important for explaining additional events that are not described only by the QE and RES channels.

The contribution of each type of interaction is critically important for data interpretation, and the measurements themselves are used to accurately fit models and reduce uncertainties in the physical parameters of neutrino oscillations [3].

### 2.3 Weak interaction

Quasi-elastic interactions of neutrinos with nucleons represent one of the fundamental processes in neutrino physics, which plays a key role in the study of neutrino oscillations and the determination of mixing parameters. These interactions are especially important in accelerator experiments, where information about the characteristics of the neutrino beam is reconstructed based on the distribution of quasi-elastic scattering events in the near and far detectors. Therefore, high accuracy of the theoretical description of such processes is a prerequisite for the correct interpretation of experimental data.

When describing the weak interaction between fermions, especially in systems with an internal structure such as nucleons, it should be borne in mind that particles are not point objects. Their electromagnetic and weak properties depend on the distribution of charge and spin, which is taken into account through form factors — functions depending on the square of the transmitted four-pulse  $Q^2$ . Of particular interest in this context is the axial form factor  $F_A(Q^2)$ , which describes the distribution of the spin (axial) charge in a nucleon. It is customary to parametrize it in the form of a dipole function [4]:

$$F_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}, \quad (1)$$

where  $g_A \approx -1.27$  is the axial charge known from  $\beta$ -decay, and  $M_A$  is the axial mass of the nucleon, which determines the scale of change in the form factor with increasing  $Q^2$ .

Axial currents are not a purely theoretical construction; they reflect the fundamental nature of the weak interaction and determine the nature of the interaction of neutrinos with matter. The value of  $M_A$ , obtained from experimental data, has a noticeable error, which leads to significant uncertainty in the calculations of cross-sections of QE processes. This, in turn, affects the accuracy of the reconstructed neutrino oscillation parameters and limits the possibilities of modern accelerator experiments.

In this paper, we consider an alternative approach to modelling QE neutrino scattering in a nuclear medium based on the relativistic Fermi gas model (RFG). The key novelty of the approach is the refined interpretation of the axial mass of the nucleon, aimed at reducing the uncertainty associated with it. This is achieved through a more realistic description of nuclear effects and interaction dynamics, which significantly improves the accuracy of cross-sectional predictions and increases the reliability of theoretical predictions.

### 3 A model with a running axial mass

The running axial mass model [5] is an approach developed to solve a fundamental problem in neutrino physics - the inconsistency of experimental measurements of the axial mass of a nucleon in various energy ranges. It is important to note that the  $M_A^{run}$  model is purely phenomenological and is not based on a first-line theory. It is an effective way to parametrize complex nuclear effects that are difficult to account for in simple models.

Experiments show that the effective axial mass  $M_A$  systematically changes with the neutrino energy. Low-energy experiments such as MiniBooNE require significantly higher values of  $M_A \approx 1.35$  GeV, while high-energy experiments such as NOMAD are consistent with the standard value of  $M_A \approx 1.05$  GeV. Also, the axial mass values extracted from experiments with heavy nuclear targets at low energies are in conflict with the formal global average value of  $M_A = 1.026 \pm 0.021$  GeV, obtained from experiments with light targets.

The discrepancies are explained by complex nuclear effects that are not accounted for in the standard relativistic Fermi gas model (RFG). These effects include:

- Long-lasting random-phase approximation (RPA) correlations
- Multi-nucleon interactions (2p2h mechanisms)
- Meson exchange currents (MEC)
- Short radius correlations between nucleons

Instead of detailed modeling of all microscopic nuclear effects, a simple phenomenological parametrization was proposed, which effectively reproduces the observed energy dependence through the introduction of an energy-dependent "running" axial mass  $M_A^{run}(E_\nu)$ :

$$M_A^{run}(E_\nu) = M_0 \left(1 + \frac{E_0}{E_\nu}\right) \quad (2)$$

where:

- ▶  $M_0 = 1.008 \pm 0.025$  GeV is the asymptotic value of the axial mass at high energies. It represents the "true" axial mass of a nucleon in the high-energy range, when nuclear effects can be ignored.
- ▶  $E_0 = 331_{-54}^{+57}$  MeV - the energy parameter characterizes the energy scale in which nuclear effects are manifested. Its magnitude, on the order of the binding energy of the nucleons in the nucleus, indicates a connection with the nuclear structure.
- ▶  $E_\nu$  is the neutrino energy in a laboratory system

At high energies ( $E_\nu \gg E_0$ ), the model returns to the standard value:  $M_A^{run} \rightarrow M_0 \approx 1.0$  GeV, which is consistent with deuterium measurements and high-energy experiments. At low energies ( $E_\nu \ll E_0$ ), the axial mass increases significantly:  $M_A^{run} \approx M_0 (E_0/E_\nu)$  which phenomenologically takes into account the enhancement of the cross section due to multinuclear mechanisms. The model uses the well-known correlation between the mean square of the transmitted pulse  $\langle Q^2 \rangle$  and the neutrino energy to transition from the  $M_A^{run}(Q^2)$  dependence to the simpler  $M_A^{run}(E_\nu)$  dependence.

## 4 NOvA

NOvA (NuMI Off-Axis  $\nu_e$  Appearance) [2] is a long-baseline neutrino experiment aimed at studying neutrino oscillations. The experiment is being carried out under the supervision of Fermilab (USA), one of the leading centers in the field of high energy physics. NOvA consists of two detectors: the Near Detector, located on the Fermilab site in Batavia, Illinois, and the Far Detector, located in Ash River, Minnesota.

The NOvA experiment represents one of the key directions in modern neutrino physics, combining large-scale engineering implementation, advanced methodology, and outstanding scientific potential. The main task of NOvA is to study neutrino oscillations, in particular, the transition of muon neutrinos ( $\nu_\mu$ ) to electron neutrinos ( $\nu_e$ ). NOvA provides critical data to refine the parameters of the neutrino mixing matrix (Pontecorvo–Maki–Nakagawa–Sakata matrices, PMNS), including the angle  $\theta_{13}$ , to study the mass hierarchy of neutrinos and possible CP violation in the lepton sector. However, the contribution of this experiment goes far beyond the scope of oscillatory physics. NOvA plays a primary role in precision studies of neutrino-nuclear interactions, especially in the 1-5 GeV energy range, where the main difficulties of theoretical description are concentrated.

The experiment uses a NuMI neutrino beam (Neutrinos at the Main Injector) created by the Fermilab accelerator. Protons with an energy of 120 GeV bombard a graphite target, resulting in the formation of pions and kaons, which, in turn, decay into muons and neutrinos. With the help of magnetic focusing systems, a predominantly muon neutrino beam is formed. The beam is directed to a small angular displacement (off-axis by 14 mrad), which provides a narrow neutrino energy spectrum with a maximum of about 2 GeV — near the maximum oscillation probability.

The Near Detector (290 tons), located 1 km from the accelerator at a depth of about 100 m, measures the composition of the beam before the start of oscillations, which is important for calibration and reducing systematic uncertainties. The Far Detector (14 kilotons) is located on the surface 810 km from the beam and registers oscillating neutrinos. Both detectors are based on the principle of a fully active segmented structure consisting of plastic extruded cells (PVC) filled with a liquid scintillator. The unique geometry of the detectors provides three-dimensional reconstruction of particle tracks with high spatial resolution. Each cell (at the near detector) has a cross-section of  $3.9 \times 6.6 \text{ cm}^2$  and a length of 4.0 m, which makes it possible to accurately measure the energy and direction of secondary particles formed during neutrino interactions.

It is critically important to measure cross sections of charged current interactions for muon and electron neutrinos in order to correctly interpret oscillation data, build accurate models of neutrino interactions, and calibrate event generators (GENIE, NEUT, etc.) used in all neutrino experiments. The main processes of interest are quasi-elastic scattering (QE), resonant and deep elastic interactions (RES, DIS), as well as neutral-current processes. The NOvA near detector plays a key role in these measurements, as it provides data on all types of interactions. An important task is also the separation of events with overlap — when several neutrinos interact in the same time window — and the correct determination of the neutrino energy from the observed interaction products.

The double differential cross sections ( $d^2\sigma/dTdcos\theta$ ) are measured directly from the near detector data for different energies and angles of lepton emission. Accurate cross-section measurements help

significantly reduce systematic uncertainties and contribute to the development of theoretical models included in neutrino generator GENIE used in simulations. The experiment also allowed us to test and adjust models that take into account complex nuclear effects, RPA, 2p2h, etc.

NOvA's unique contribution is the large statistics on the  $\nu_e$  cross-section, which is extremely difficult for other experiments with accelerator beams, but is very important for future precision measurements of CP violation.

## 5 Results

### 5.1 Data

As part of the program, the theoretical data of the running axial mass model obtained using the GENIE neutrino event generator were compared with the data of muon neutrino interaction cross sections from the NOvA near detector [6].

GENIE Neutrino Event Generator (Generates Events for Neutrino Interaction Experiments) [7] was chosen because it is a universal software tool for modeling neutrino interactions with nuclei. GENIE implements statistical modeling of neutrino interactions based on the Monte Carlo method. Starting with GENIE v3.4.0, a running axial mass model is built into the generator (denoted as  $M_A^{run}$  Axial-FormFactorModel). The integration of the  $M_A^{run}$  model into GENIE ensures its direct application in the full simulation chain, including event generation, simulation of their passage through the detector, and subsequent reconstruction.

The experimental data were taken from open sources of the Fermi Laboratory. The signal definition for this analysis is the true interactions of muon neutrinos with the vertex in the reference volume of the NOvA near detector, that is, in the central region measuring 2.7 m x 2.7 m x 9.0 m. The double differential cross section, depending on the muon kinematics, is estimated in a limited phase space:  $0.5 < \cos \theta < 1.0$ ,  $0.5 \text{ GeV} < p < 2.5 \text{ GeV}$ . The only differential cross-section depending on the square of the momentum transfer ( $Q^2$ ) and the neutrino energy ( $E_{\nu}$ ) is calculated only in the limited phase space defined above. The ROOT file contains the results of single ( $Q^2$  and  $E_{\nu}$ ) and double differential cross sections (MuKin), general taxonomy, shape-only taxonomy, and statistical covariance matrices, and a covTo2DBin histogram containing the transformation from covariance matrix binning to 2D binning for muon kinematics measurements.

### 5.2 Comparison methods

#### 1. Visual (graphical) comparison

At the first stage, the theoretical curves (or histograms) are visually compared with the experimental data presented on the graphs. This method allows you to quickly assess the overall quality of the model, but does not provide a quantitative measure of agreement. It allows you to quickly identify obvious discrepancies: systematic underestimation or overestimation of predictions in certain kinematic areas, for example, at large scattering angles or high muon energies. However, visual assessment is subjective and does not provide a numerical measure of accuracy, so it serves only as a starting point for a more rigorous analysis.

For an objective quantitative assessment, three variants of the chi-square criterion are used [8].

#### 2. Diagonal $\chi^2$ ( $\chi_{diag}^2$ )

Formula:

$$\chi_{diag}^2 = \sum_{i=1}^N \frac{(E_i - T_i)^2}{\sigma_i^2} \quad (3)$$

- $E_i$  – experimental value in the  $i$ th bin,
- $T_i$  – theoretical prediction,

- $\sigma_i$  – total measurement error (statistical + systematic).

It can greatly underestimate the actual errors and lead to incorrect conclusions. Using this method alone is inadequate due to the high correlation of the data. A critical disadvantage of this method is the complete disregard of correlations between different data bins. Systematic errors, such as the uncertainty of the neutrino flux, can be strongly correlated between many bins. Neglecting these correlations leads to a catastrophic underestimation of the actual error. Thus,  $\chi_{diag}^2$  can be seriously misleading about the quality of the model.

### 3. Total $\chi^2$ ( $\chi_{tot}^2$ ) taking into account the full covariance matrix

Formula:

$$\chi_{tot}^2 = (E - T)^T W_{tot}^{-1} (E - T) \quad (4)$$

- E, T are vectors of experimental and theoretical values,
- $W_{tot}$  is a complete covariance matrix that includes all error sources and their correlations.

It takes into account all known correlations between bins (both statistical and systematic). The most objective measure of the model's agreement with the data. Its calculation involves not just a set of errors, but the full covariance matrix  $W_{tot}$ . This matrix contains not only the individual variances (error squares) for each group, but also the covariances between all pairs of groups, which show how much they are biased relative to each other due to common systematic errors. This method provides the most objective and reliable measure of the general agreement of the model with the data, as it takes into account all known information about experimental uncertainties and their interdependencies. Systematic errors are dominant and highly correlated. Using the inverse covariance matrix in the formula of the full  $\chi_{tot}^2$  allows us to correctly account for these correlations.

### 4. Shape $\chi^2$ ( $\chi_{sh}^2$ ) with normalization separation

Formula:

$$\chi_{sh}^2 = (E - NT)^T W_{sh}^{-1} (E - NT) + \frac{(N - 1)^2}{\delta^2} \quad (5)$$

- E, T are vectors of experimental and theoretical values,
- $W_{sh}$  is a covariance matrix that excludes the contribution of general normalization,
- N is a normalization factor chosen so as to minimize  $\chi^2$ ,
- $\delta$  is the average relative error of normalization.

Allows you to evaluate how well the model describes the shape of the distribution, regardless of the overall absolute normalization. The normalization factor N is selected during the minimization process. It is often useful to evaluate how well the model describes exactly the shape of the distribution, regardless of the possible constant multiplier (normalization). The statistics of  $\chi_{sh}^2$  (chi-square shape) are used for this. In this approach, the theoretical prediction T is multiplied by a normalization factor N, which is selected in such a way as to minimize the discrepancy in shape. The formula  $\chi_{sh}^2$  itself includes two terms: the first evaluates the agreement in form using the covariance matrix  $W_{sh}$  (from which contributions related to the general normalization are excluded), and the

second is the penalty term  $(N - 1)^2/\delta^2$ , which corrects when the normalization of  $N$  deviates from unity. The value  $\delta$  represents the average relative uncertainty of data normalization (for NOvA  $\delta \approx 11.2\%$ ). This method is especially important for comparing models that can predict the shape well, but require a little renormalization for best agreement. The  $\chi_{sh}^2$  method is an elegant solution to this problem. By introducing a normalization factor  $N$  and adding the penalty term  $(N - 1)^2/\delta^2$ , the method explicitly separates the error in normalization from the error in the form. The penalty term acts as an "a priori" constraint on  $N$  based on an independent estimate of the relative normalization error  $\delta$ .

### 5.3 Accomplished work

As part of the summer research program, extensive and comprehensive work was carried out on elementary particle physics. The program consisted of a complete scientific cycle: from mastering the fundamentals to analyzing real experimental data. The work began with a deep dive into the theoretical foundations. The nature of neutrinos, their properties, types of interactions, and their role in the Standard Model were studied in detail. This made it possible to clearly identify the range of the most pressing and unresolved problems in this area. The topic of neutrino interaction cross sections was chosen [6]. A review of the literature on the chosen topic was conducted. The central element of theoretical training was the study of modern theoretical models describing the interaction of neutrinos with matter. In this context, the running axial mass model has been thoroughly studied. This review provided a critical foundation for further work. The next stage was to get acquainted with the practical side of research. The principles of the NOvA experiment are considered. Its unique features were analyzed in detail: from the tasks and measured data to the location and design of the detectors [9]. Special methodological attention was paid to the study and development of tools for comparing experimental data with theoretical models and choosing the optimal one. The criteria of  $\chi^2$  (chi-square) were chosen.

The final and most fascinating part of the work was devoted to the analysis of real data obtained by the NOvA collaboration. The transition from theory to practice was embodied in the creation of a specialized C++ program using the ROOT framework [10], which is a standard for data processing in high-energy physics. The written program performed a key task: the automated calculation of the agreement coefficients  $\chi^2$  between theoretical predictions and experimental data. The program also generated a series of graphs where the experimental spectrum with its statistical errors was superimposed on the curve of theoretical expectations. This visual representation, supplemented by calculated agreement coefficients, made it possible not only to verify the discrepancy, but to conduct a deep qualitative and quantitative analysis of it, to see in which areas of energy the model with a running axial mass demonstrates a statistically significant advantage, and where its predictions coincide with the Standard Model.

### 5.4 Results of work

Graphs were plotted for  $\sigma(E_\nu)/E_\nu$  and  $\frac{d\sigma}{dQ^2}$ . Two types of data were presented on the common axes: experimental values depicted as a set of red dots, each point accompanied by an error visualized by a vertical segment (error bar), and a theoretical curve represented as a blue line.

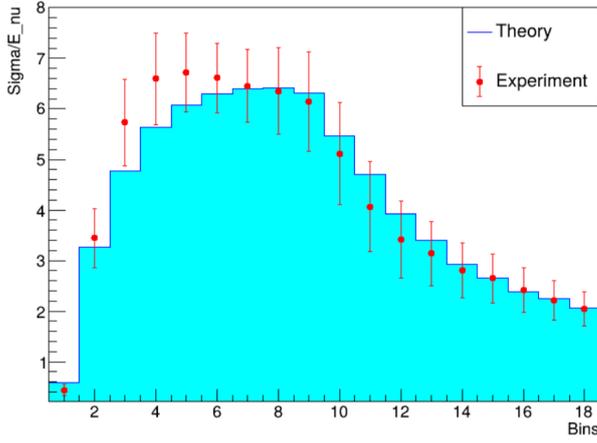


Figure 1: Comparison of experimental and theoretical data for  $E_\nu$  cross sections

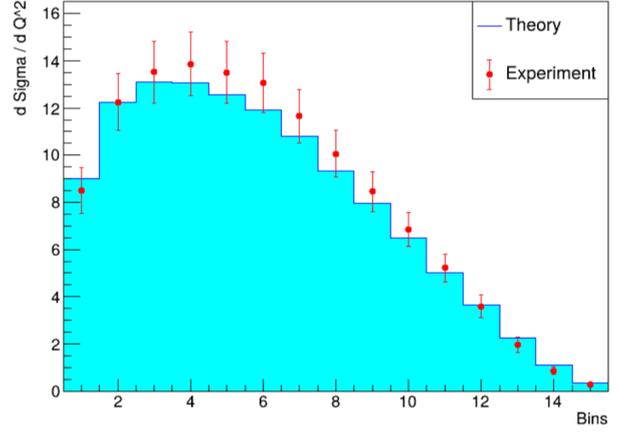


Figure 2: Comparison of experimental and theoretical data for  $Q^2$  cross sections

All 3 coefficients  $\chi^2$  were calculated for  $\frac{\sigma(E_\nu)}{E_\nu}$ .

$$\chi_{diag}^2 = 6.54408$$

$$\chi_{tot}^2 = 33.6827$$

$$\chi_{sh}^2 = 33.0218, N=1.021$$

For  $\frac{d\sigma}{dQ^2}$  all 3 coefficients of  $\chi^2$  were also calculated.

$$\chi_{diag}^2 = 6.9874$$

$$\chi_{tot}^2 = 33.2055$$

$$\chi_{sh}^2 = 36.7079, N=1.115$$

For  $\frac{d^2\sigma}{d\cos\theta_\nu dT_\mu}$ , only the diagonal coefficient was calculated.

$$\chi_{diag}^2 = 124.586$$

## 5.5 Analysis and interpretation

A comparison of experimental NOvA data with the predictions of the running axial mass model shows different levels of agreement depending on the analyzed value.

A low value of  $\chi_{diag}^2$  could indicate a good agreement when only individual point errors are taken into account, but this is a dangerous misconception. This value does not take into account correlations between bins and systematic errors.

The real value of  $\chi_{tot}^2$  is about 5 times more. This indicates the presence of strong positive correlations between measurements in different energy bins due to common systematic uncertainties (for example, an error in determining the neutrino flux). A high value of  $\chi_{tot}^2$  indicates that the model does not perfectly describe the data when these correlations are taken into account.

The value of  $\chi_{sh}^2$  is very close to  $\chi_{tot}^2$ . This means that the main part of the discrepancy falls precisely on the form of the dependence of  $\sigma/E_\nu$  on energy, and not on the general normalization. The normalization factor  $N = 1.021$ , taken from the article, is very close to unity, which indicates that the model almost correctly predicts the absolute value of the cross section. The problem is that the form of energy dependence predicted by the model does not fully match the data.

For the distribution of  $\sigma(E_\nu)/E_\nu$ , the number of bins is 18. For  $\chi_{tot}^2 \approx 33.7$  and  $N = 18$ , we get

$\chi^2/\nu \approx 1.9$ . This value is higher than one, which indicates noticeable discrepancies, but they are still within the statistically acceptable level. In the case of  $\chi_{diag}^2 \approx 6.5$ , the normalized metric would be only  $\chi^2/\nu \approx 0.36$ , which clearly underestimates the actual voltage and confirms that the diagonal estimate does not reflect the actual situation.

In the case of the  $d\sigma/dQ^2$  distribution, the situation is more tense. There is also a sharp increase in statistics from the diagonal value to the full value, which underlines the critical role of inter-spin correlations.

The key difference from the previous case is that the value of  $\chi_{sh}^2$  exceeds  $\chi_{tot}^2$ , and the normalization factor  $N = 1.115$  deviates significantly from unity. This is an important indication of the nature of the discrepancy. To improve the agreement in form ( $\chi_{sh}^2$ ), the model needs to increase its prediction by  $\sim 11.5\%$ . However, this worsens the general agreement ( $\chi_{tot}^2$ ), since the data does not require such a large renormalization — their absolute normalization is better described by the original model ( $N = 1.0$ ). This indicates a compromise between form and normalization. A model cannot perfectly describe both at the same time. The most likely reason is that the model somewhat distorts the shape of the dependence  $d\sigma/dQ^2$ , and to partially compensate for this distortion, the minimization algorithm  $\chi_{sh}^2$  is forced to apply significant renormalization.

For the distribution of  $d\sigma/dQ^2$ , for  $\chi_{tot}^2 \approx 33.2$  and 15 bins, we get  $\chi^2/\nu \approx 2.2$ . The shape-only value is even higher:  $\chi_{sh}^2 \approx 36.7$ , which gives  $\chi^2/\nu \approx 2.4$ . This is already a noticeable strain, indicating that the model does not reproduce the shape of the distribution with the accuracy that is expected at the level of experimental errors.

For the double differential value  $d^2\sigma/(d\cos\theta_\mu dT_\mu)$ , it was possible to calculate only the diagonal statistics. The value of  $\chi_{diag}^2$  for 158 bins is quite high, but without taking into account correlations, it does not give a correct idea of agreement. We can extrapolate the conclusions from previous analyses and assume that  $\chi_{tot}^2$  will be many times higher when correlations are taken into account. This is the most difficult distribution to describe, since it is two-dimensional and sensitive to the greatest number of aspects of theory (impulse distributions of nucleons in the nucleus, transverse response, end-state processing, etc.). In 2D distributions, inter-spin correlations associated with migration of events between cells according to muon kinematics can be very strong, and their ignoring artificially inflates discrepancies. Therefore, this result should be considered only as a rough estimate; for a physically meaningful conclusion, it is necessary to use the full or shape-only covariance matrix.

## 6 Conclusion

As part of the summer educational program, a study was conducted on the comparative verification of the agreement of experimental data on inclusive neutrino-nucleon interaction cross-sections in the range of GEV energies of muon neutrinos with the theoretical predictions of the running axial mass model ( $M_A^{run}$ ). The object of the study was the data obtained on the near detector of the NOvA experiment, and the phenomenological model with a running axial mass ( $M_A^{run}$ ) was used as a theoretical model. The main method of analysis was the application of a statistical approach using the chi-square criterion, including the calculation of diagonal ( $\chi_{diag}^2$ ), full ( $\chi_{tot}^2$ ) and form factor ( $\chi_{sh}^2$ ) statistics, taking into account full covariance matrices, which allowed us to correctly account for systematic errors and correlations between the data.

The analysis showed that the  $M_A^{run}$  model reproduces the general scale of integral quantities and demonstrates a limited but competitive agreement with experimental data. Qualitatively, the model satisfactorily reproduces the main features of all three studied distributions: the double differential cross section  $d^2\sigma/(d\cos\theta_\mu dT_\mu)$ , the differential cross section  $d\sigma/dQ^2$ , and the ratio of the total cross section to energy  $\sigma(E_\nu)/E_\nu$ . This indicates its fundamental suitability for use as a data analysis tool, in particular, for modeling the background and signal in the long-range NOvA detector when studying neutrino oscillations.

However, at the level of a detailed description of the distributions of kinematic variables, quantitative analysis revealed systematic discrepancies. The most significant deviations were observed in the description of the shape of the distributions, especially depending on the square of the transmitted 4-pulse,  $Q^2$ . The calculated chi-square values significantly exceed the values of the diagonal statistics, which clearly indicates the presence of strong correlated systematic shifts that are not taken into account by the simplified approach. The tension between optimal normalization and the description of the shape of the distribution indicates that the model cannot accurately predict both the absolute scale and the kinematic dependence of the section at the same time. This indicates that the main problem of the model lies not only in the general normalization, but also in the accurate transmission of the  $Q^2$  dependence.

The physical reason for the identified discrepancies is most likely the simplifications embedded in the  $M_A^{run}$  model, which empirically, through the introduction of an energy-dependent running axial mass, tries to take into account complex nuclear effects such as the contribution of meson exchange currents (MEC), multi-nucleon correlations (2p2h), random phase approximation (RPA) and interactions in the final state (FSI). These effects require a more fundamental microscopic description. In addition, models of other processes (resonant generation, deeply inelastic scattering) implemented in the GENIE generator contribute to the overall uncertainty.

Thus, the  $M_A^{run}$  model, being easy to implement and consistent with the accelerator experiment, confirms its applicability in analyzing data from neutrino oscillation experiments. But in order to achieve the accuracy required by next-generation experiments (DUNE, Hyper-K), the model can be further developed. In the future, the development of the work will be associated with further verification of the model with  $M_A^{run}$ . In particular, the  $M_0$  and  $E_0$  parameters can be adjusted by conducting a global fit on an expanded set of experimental data, including the results of not only NOvA, but also other collaborations such as MINERvA, T2K, and MicroBooNE, which will increase the versatility and

predictive power of the model. An important step will be to carry out a separate fit of the model parameters for different ranges of neutrino energies, which can reveal the features of the model's operation in different modes of interaction and more accurately localize the sources of discrepancies between theory and experiment. An additional area of future work is the comparison of the interaction cross sections of electron and muon neutrinos. Of particular interest is the analysis of differences in the axial form factor  $F_A$ , which may differ depending on the lepton channel and, consequently, affect the accuracy of the data description. A systematic comparison of the factors will clarify the applicability of the model with  $M_A^{run}$  to various lepton flavours and expand its versatility in describing cross-sections of neutrino-nucleon interactions. The upcoming data on different flavours of antineutrinos, which are being actively collected in the framework of modern experiments, seem promising. Comparing the model with these data will be crucial for testing the hypothesis of CP violation in the lepton sector and searching for traces of new physics. The availability of data on muon and electron antineutrinos will allow for a direct comparison: to check whether the model with a running mass of  $M_A^{run}$  predicts the correct cross-sections for these processes. This will be a key test for the universality of the model and its ability to become a single effective tool for predicting cross-sections in a wide variety of neutrino and antineutrino interactions.

To summarize, the work not only allowed us to develop modern methods of statistical data analysis, but also outlined clear ways to improve theoretical models of neutrino-nuclear interactions.

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