



JOINT INSTITUTE FOR NUCLEAR RESEARCH
Veksler and Baldin laboratory of High Energy Physics

FINAL REPORT ON THE START PROGRAMME

*Usage of Lobachevsky (hyperbolic)
geometry in studies of relativistic nuclear
interactions*

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Abstract

In this paper relativistic-invariant method of investigation of relativistic nuclear interactions is considered. Lobachevsky geometry is used to describe relativistic nucleon collisions and it was shown that this method can derive more detailed information about secondary particles as well as help us discover new properties of multi-particle production process. This method was applied to the study of reaction $p(10 \text{ GeV}/c) + C$ in propane chamber LHEP and compared to the UrQMD model.

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Introduction

There are many mysteries remaining in nuclear physics. One of them is quark-gluon plasma and searches for it continue nowadays. Ongoing searches in this field use nonrelativistic-invariant quantities which are not able to fully take into account relativistic effects. Over a long period of time both theoretical and experimental studies had developed this method [1, 2, 3]. Yet it still has to be experimentally approved.

One of the relativistic effects is the existence of ultimate speed limit, equal to the speed of light. It implies that the physical velocities space is non-euclidian and it has Lobachevsky (hyperbolic) geometry. Each particle is a point in velocities space, the distance between two particles i and k is considered as a relative rapidity ρ_{ik} , which is calculated according to formula (1):

$$\rho_{ik} = \tanh^{-1} \beta_{ik} \quad (1)$$

where $\beta_{ik} = \frac{v_{ik}}{c}$ is a dimensionless relative speed of particles i and k .

If we consider three particles registered in the experiment, we can construct a rapidity triangle for them in the Lobachevsky space (fig. 1) and study its geometrical properties in the experiment.

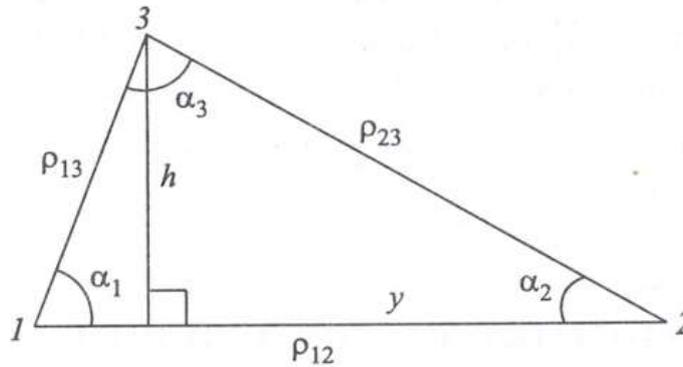


Figure 1. A scheme of the rapidity triangle for particles: 1 – projectile, 2 – target, 3 – secondary particle. The lines are drawn straight for easier perception

In order to study nuclear interactions many models are used. In this study UrQMD (Ultra-relativistic Quantum Molecular Dynamics) model [4, 5] (fig.2) was used to compare theoretical results with experimental data.

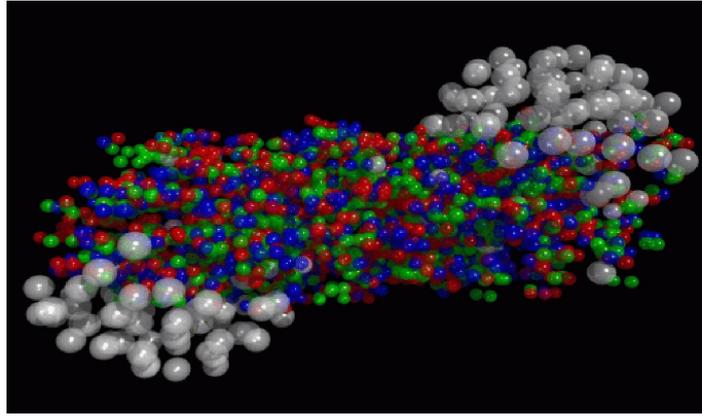


Figure 2. A scheme of the nuclear collision modeled with UrQMD

Project objectives

The use of a new method for analyzing nuclear interactions in Lobachevsky (hyperbolic) space to uncover new patterns in nuclear interactions and particle production processes, as well as to identify effects that cannot be predicted by modern models.

Methodology

Experimental data of p (10 GeV/c) + C reaction was obtained from the LHEP propane bubble chamber. Pictures of the events (fig.3) were scanned from the films and digitized in order to fit tracks using program. As a result, a binary file containing description of all registered events was formed and used in the present analysis.

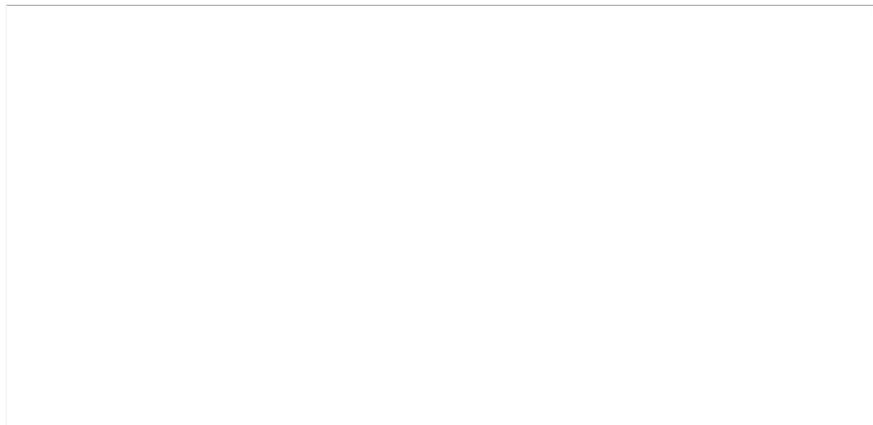


Figure 3. An example of a picture of the events registered in LHEP propane bubble chamber.

Modelled data of $p(10 \text{ GeV}/c) + C$ reaction was obtained using personal computer in the LHEP office via compiled UrQMD model. Collision impact parameter was set random in range from 0 to 2.7 fm (approximately the size of C_{12} nucleus). All events without interaction were discarded and only protons and charged π mesons were used in the analysis.

Analysis was conducted using only Root CERN framework.

Results

For educational purposes, the analysis of $p(10 \text{ GeV}/c) + C$ reaction data was first performed using "classical" quantities — p , p_T and longitudinal rapidity — which are not Lorentz-invariant. The momentum (p) distribution for protons (fig. 4) exhibited a sharp peak in the region of $p \sim 1 \text{ GeV}/c$, while the distribution for π^+ mesons (fig. 5) showed a cutoff at $p \sim 1 \text{ GeV}/c$. Given the well-known issue [6] of particle identification in bubble chambers at momenta $> 1 \text{ GeV}/c$, we suppose that π^+ mesons with $p > 1 \text{ GeV}/c$ were misidentified as protons in the experiment. Taking this factor into account, the model data were also subsequently modified to describe the experimental data.

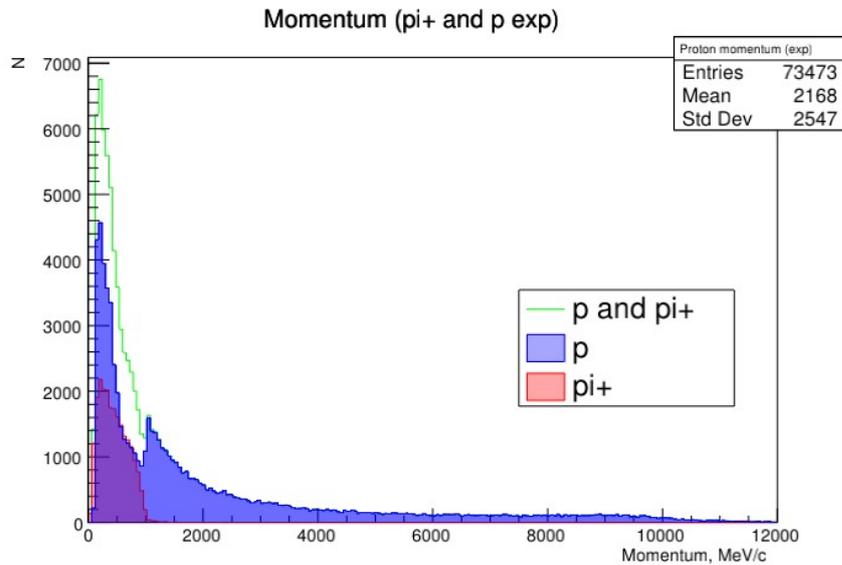


Figure 4. Experimental momentum distribution. The peak near $p = 1 \text{ GeV}/c$ in proton's momentum (blue) distribution is clearly seen.

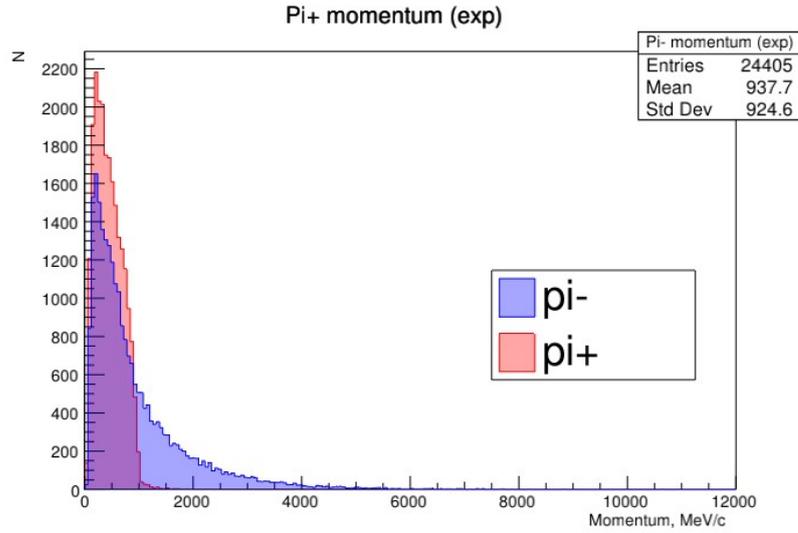


Figure 5. A comparison of experimental momentum distributions for π^+ and π^- . Distribution for π^+ (red) has a sharp cutoff at $p = 1 \text{ GeV}/c$.

Following this, analysis of data using Lobachevsky space was performed. We analysed velocities triangles for particles (fig.1) to determine if they are isosceles. For isosceles triangle: $\alpha_1 - \alpha_2 = 0$ and $\rho_{23} - \rho_{13} = 0$, i.e. we studied $\alpha_1 - \alpha_2$ and $\rho_{23} - \rho_{13}$ distributions for triangles formed of: 1 – projectile (proton 10 GeV/c), 2 – target (C_{12} nucleus), 3 – secondary particle (proton or π^-).

Distributions for triangles with π^- (fig.6, fig.7) are consistent with model, while in $\alpha_1 - \alpha_2$ distribution for triangles with protons (fig. 8) two additional peaks are seen in the regions $\alpha_1 - \alpha_2 = -1.25 \text{ rad}$ and 0.75 rad , which correspond to protons from target's fragmentation process and to scattered projectiles respectively. This explanation is supported by the $\rho_{23} - \rho_{13}$ distributions for such triangles (fig. 9).

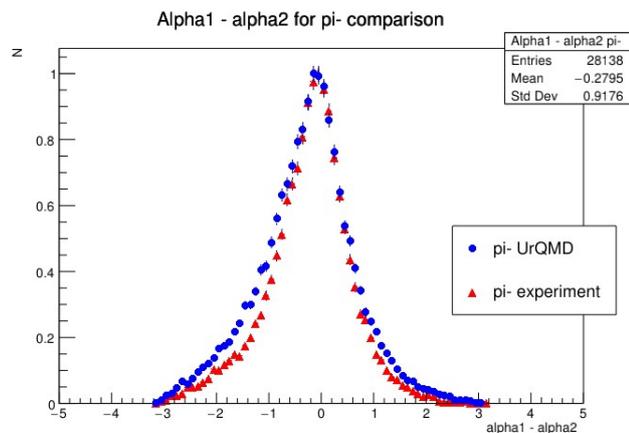


Figure 6. $\alpha_1 - \alpha_2$ distribution for triangles with registered π^- in the experiment (red) compared with π^- from model (blue).

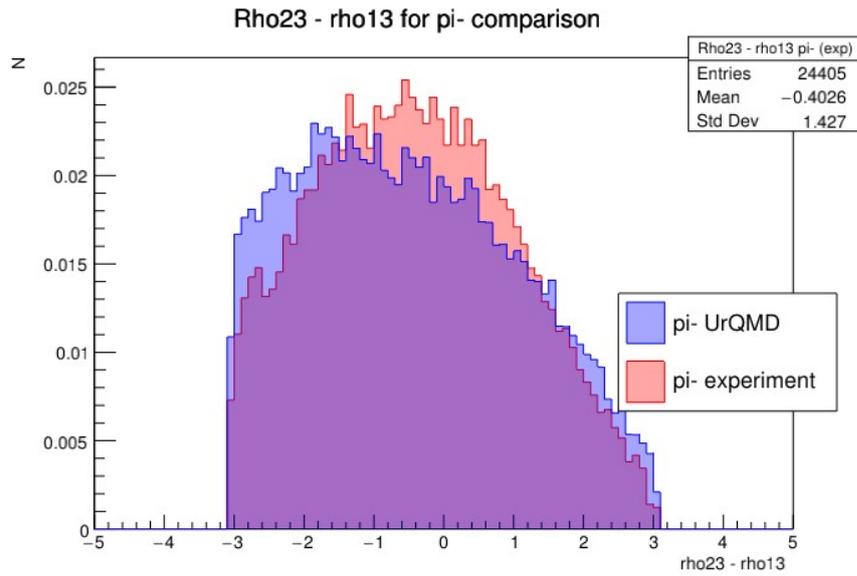


Figure 7. $\rho_{23} - \rho_{13}$ distribution for triangles with registered π^- in the experiment (red) compared with π^- from model (blue)

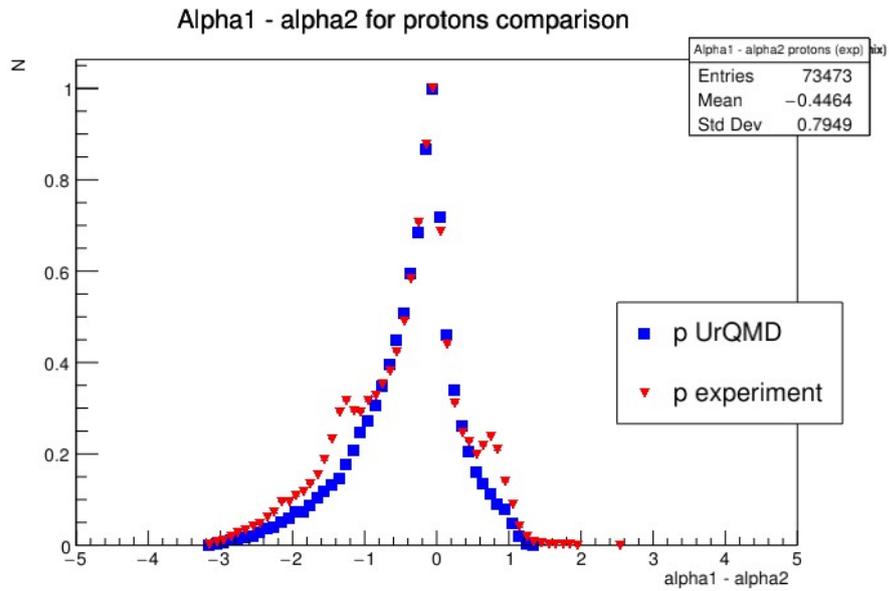


Figure 8. $\alpha_1 - \alpha_2$ distribution for triangles with registered protons in the experiment (red) compared with protons from model (blue). Two additional peaks are visible in the experimental distribution.

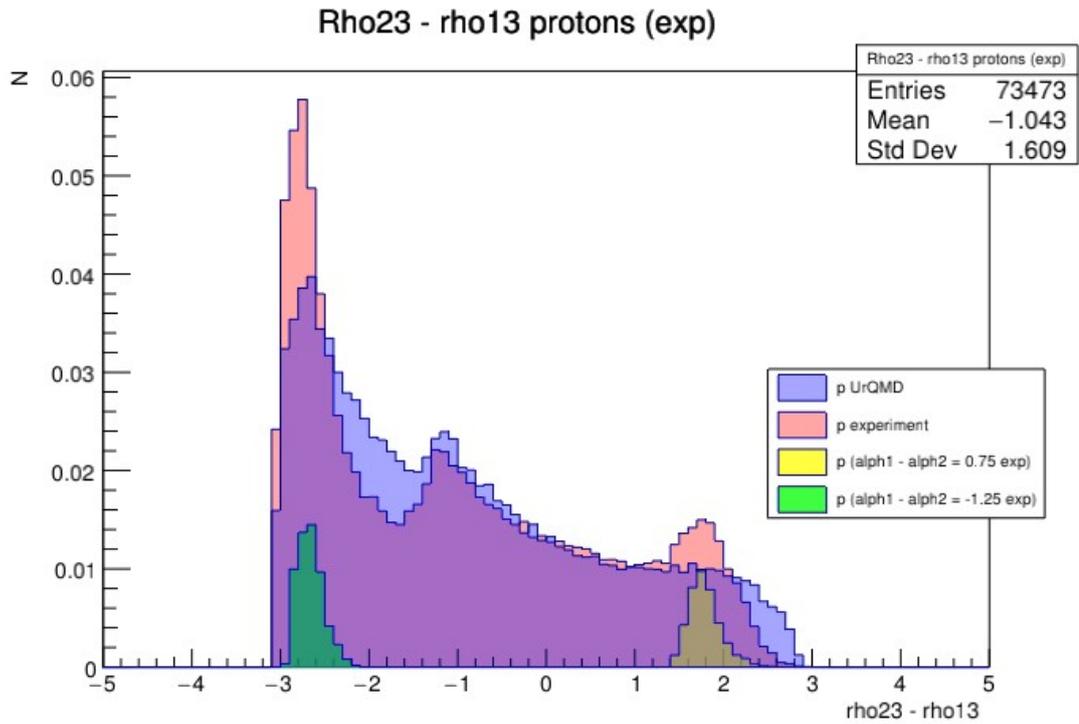


Figure 9. $\rho_{23} - \rho_{13}$ distribution for triangles with registered protons in the experiment (red) compared with protons from model (blue)

As can be seen from figure 9, the peak corresponding to $\alpha_1 - \alpha_2 = -1.25$ rad contains protons that in rapidity space are located near the target, i.e., they are its fragments. In contrast, the peak at $\alpha_1 - \alpha_2 = 0.75$ rad consists of protons lying near the incident projectile, i.e., they are scattered incident protons.

Furthermore, the characteristics of the rapidity triangle can be used to identify scattered incident protons in the experiment. As it can be seen from figure 10, protons forming triangles with an acute angle $\alpha_3 < 1.3$ rad produce a peak in the proton rapidity distribution ρ_{23} (rapidity in the laboratory frame) near value $\rho_{23} = 3$.

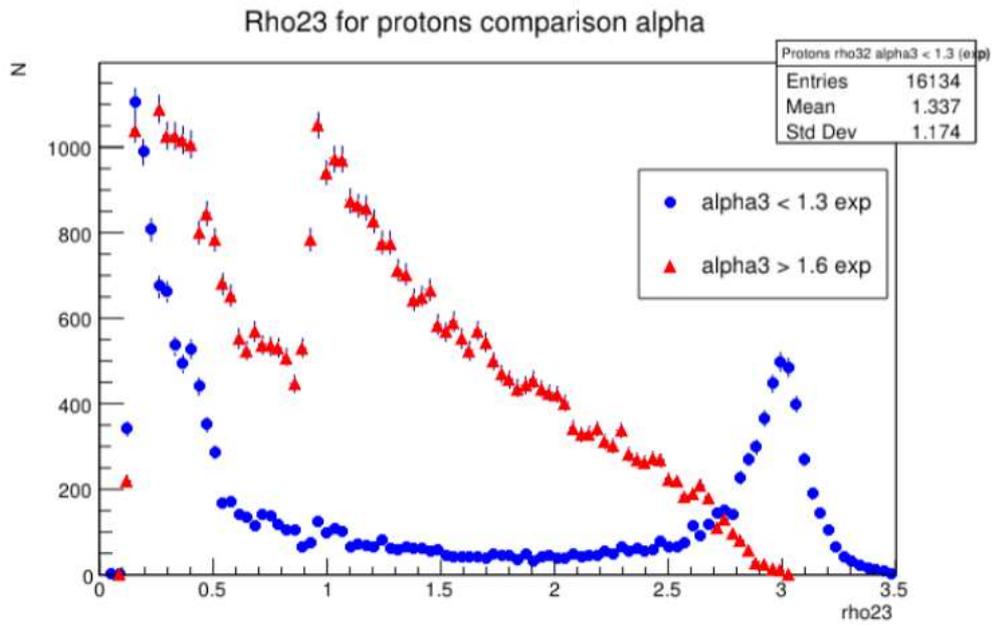


Figure 10. ρ_{23} distribution for protons in the experiment

Interestingly, the model does not reproduce this peak (fig. 11).

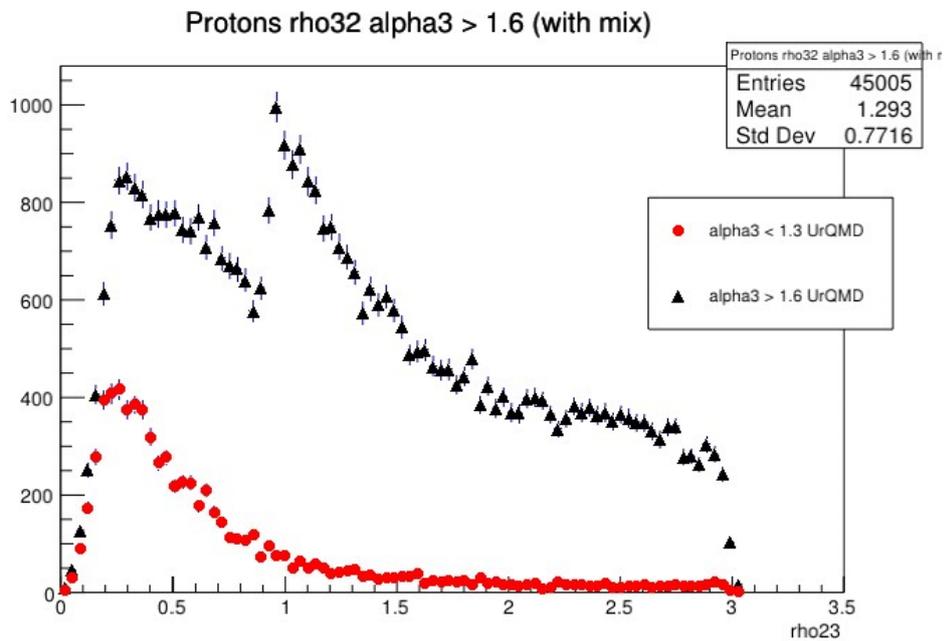


Figure 11. ρ_{23} distribution for protons in the UrQMD model.

The jump in the ρ_{23} distributions for protons near $\rho_{23} = 1$ was also analyzed. Using the model, it can be shown that this jump arises when π^+ mesons with momentum $p > 1$ GeV/c are added to the protons, as was the case in the experiment (Fig. 12).

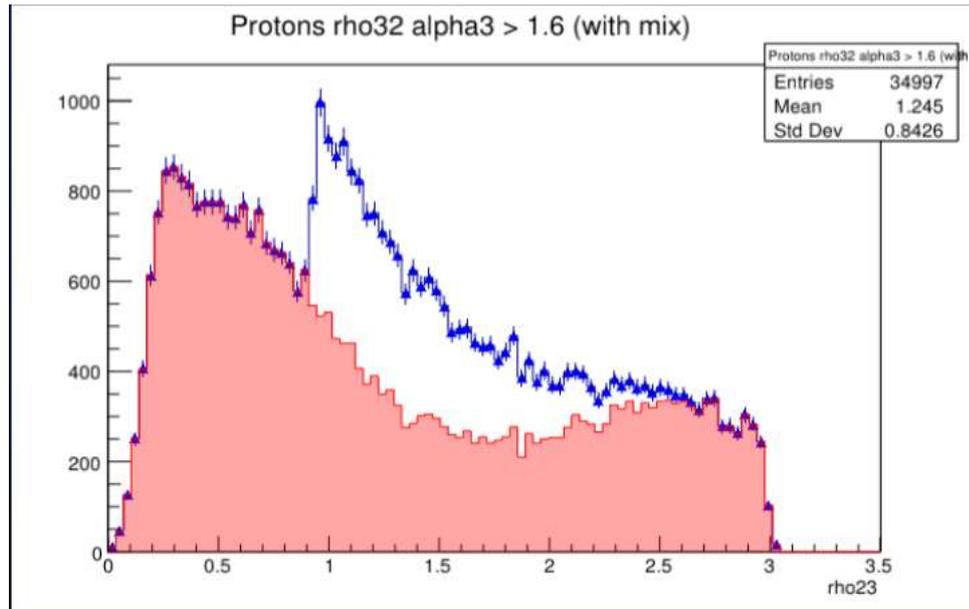


Figure 12. ρ_{23} distribution for protons with $\alpha_3 > 1.6$ in model. Blue – with addition of π^+ with $p > 1$ GeV/c. Red – without any mixing, only protons.

Based on this assumption, an attempt was made to remove this jump in the experimental distribution (fig. 13) using distribution for π^+ with $p > 1$ GeV/c from model.

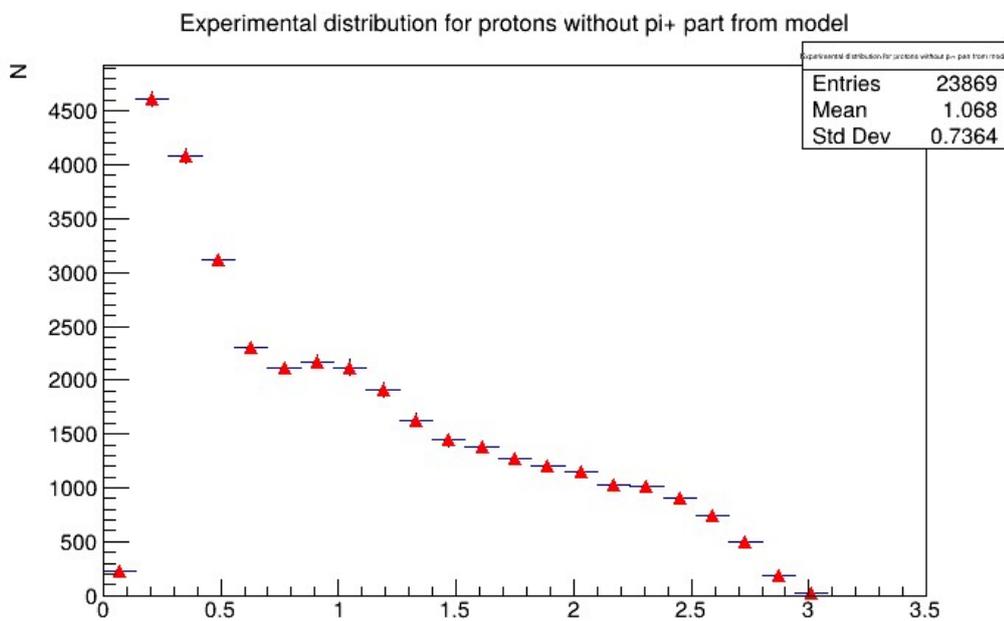


Figure 13. Experimental ρ_{23} distribution for protons with $\alpha_3 > 1.6$ after extracting π^+ with $p > 1$ GeV/c part.

Conclusion

As part of the START program internship, I gained experience in researching relativistic ion collisions, studied the fundamentals of Lobachevsky geometry and the event analysis method based on it. I also gained experience in simulating relativistic ion collisions using the UrQMD model.

As a result of this work, effects emerging in Lobachevsky space were investigated; these effects could be used in the future analysis of relativistic ion collisions at the NICA collider. Additionally, effects related to the mixing of protons and π^+ mesons — associated with the specific nature of bubble chamber experiments — were studied.

References

- 1) Черников Н. А. Геометрия Лобачевского и релятивистская механика //ЭЧАЯ.–М.: Атомиздат. – 1973. – Т. 4. – №. 3. – С. 733.
- 2) Baldin A. A. et al. Analysis of experimental data on relativistic nuclear collisions in the Lobachevsky space. – Joint Institute for Nuclear Research, Dubna (Russian Federation), 2004. – №. JINR--1-4-121-2004. – С. 7-16.
- 3) Baldin A. M. et al. Relativistic nuclear physics: Relative 4-velocity space, symmetries of solutions, correlation depletion principle, similar attitude, intermediate asymptotics. – Laboratory of High Energies, Joint Institute for Nuclear Research, Dubna (Russian Federation), 1997. – №. JINR-R--2-97-309.
- 4) S. A. Bass, M. Belkacem, M. Bleicher, M. Brandstetter, L. Bravina, C. Ernst, L. Gerland, M. Hofmann, S. Hofmann, J. Konopka, G. Mao, L. Neise, S. Soff, C. Spieles, H. Weber, L. A. Winckelmann, H. Stoecker, W. Greiner, Ch. Hartnack, J. Aichelin and N. Amelin: Microscopic Models for Ultrarelativistic Heavy Ion Collisions Prog. Part. Nucl. Phys. 41 (1998) 225-370
- 5) M. Bleicher, E. Zabrodin, C. Spieles, S.A. Bass, C. Ernst, S. Soff, L. Bravina, M. Belkacem, H. Weber, H. Stoecker, W. Greiner: Relativistic Hadron-Hadron Collisions in the Ultra-Relativistic Quantum Molecular Dynamics Model J. Phys. G: Nucl. Part. Phys. 25 (1999) 1859-1896
- 6) Armutliiski D. D. et al. Investigation of pc interaction at $p(p) = 10\text{-gev/c}$ with the emission of cumulative neutral pions. – 1986. – №. JINR-P-1-86-459.