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FINAL REPORT ON THE START PROGRAMME

Analysis of chromatic aberration of betatron oscillations

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Abstract

Achieving design parameters in complex accelerator systems, such as NICA, is fundamentally important for successful scientific research in the field of elementary particle physics. Ensuring stable and high-precision operation of accelerators requires meticulous tuning and optimization of all system components.

The development of mathematical models describing particle dynamics in accelerators is of particular importance, as it enables the prediction of particle beam behavior and identification of factors influencing their motion. Equally critical is the analysis of data from diagnostic equipment, which ensures monitoring of beam characteristics and timely detection of potential anomalies.

This research focuses on the development and refinement of mathematical models for particle dynamics in accelerators, as well as the analysis of signals from diagnostic systems for deeper understanding and optimization of the NICA accelerator complex. The implementation of these tasks contributes to ensuring reliable accelerator operation and successful execution of scientific programs.

1. Introduction

The synchrotron is one of the primary types of cyclic particle accelerators, having evolved into a key instrument for both high-energy physics and the generation of unique synchrotron radiation. Compared to simpler accelerators like the betatron, the synchrotron features a significantly more complex and flexible design, enabling it to achieve record energies — up to 6.5 TeV for protons (the Large Hadron Collider, LHC) and over 100 GeV for electrons (the Large Electron-Positron Collider, LEP). Its fundamental principle is based on the precise synchronization of a time-varying guiding magnetic field, which keeps particles on an orbit of constant radius, and a high-frequency electric field that provides acceleration. This review covers the basic operating principles of the synchrotron, details of its construction, main areas of application, and a critical analysis of a fundamental beam dynamics challenge (chromaticity) along with modern methods for its compensation, which are of particular interest for the design of next-generation facilities.

The core operating principle of a synchrotron is to maintain a constant circular trajectory for particles as their energy and speed continuously increase. The key to this is the precise synchronization of two processes: the gain in particle energy each time it passes through an accelerating element must be accompanied by a proportional increase in the strength of the magnetic field in the bending magnets. This increasing magnetic field compensates for the growing momentum of the particles, forcing them to follow the same predetermined orbit despite their rising energy. If the magnetic field remained constant, as in some other accelerator types, the accelerating particles would simply no longer fit within the vacuum chamber and would be lost. Thus, the heart of a synchrotron is a precision control system that maintains a "rigid link" between the growing power of the accelerating electric field and the increasing strength of the confining magnetic field. Acceleration is provided by the electric field of radio frequency (RF) resonators, which particles cross multiple times (up to ~10^6 times per second), receiving a small packet of energy each time.

Structure and key components of a modern synchrotron constitute a complex engineering system:

1. Magnetic System: Includes dipole (bending) magnets that define the equilibrium orbit; quadrupole magnets that provide strong beam focusing by creating a alternating field gradient; as well as sextupole and octupole magnets for correcting nonlinear effects and chromatic aberrations.
2. Vacuum Chamber: Maintains an ultra-high vacuum (on the order of 10^{-9} Torr) to minimize particle scattering on residual gas.
3. RF System: Resonators that create the accelerating electric field with strict control over amplitude and phase.
4. Injection and Extraction Systems: Include fast-acting magnets (kickers) and separators for injecting the beam into the ring and extracting it onto targets or experimental stations.
5. Synchrotron Light Sources: In specialized facilities (synchrotron light sources), devices called undulators and wigglers are installed in straight sections to generate intense, coherent radiation for user experiments.

Areas of Application for synchrotrons are extremely broad and extend far beyond fundamental physics. Powerful synchrotron radiation (SR), with its high brightness, collimation, and tunability, has become an indispensable tool:

- In Biomedicine and Pharmacology: X-ray crystallography of proteins and viral particles at SR facilities is the foundation for rational drug and vaccine design, helping to reduce development time and costs by approximately 1.5 times. Phase-contrast tomography and microscopy techniques allow the study of soft tissues and bone microarchitecture with unprecedented resolution. A promising direction is Microbeam Radiation Therapy (MRT) for treating resistant forms of cancer.
- In Materials Science and Nanotechnology: SR is used to study the electronic structure, defects, and stresses in materials at the atomic level, which is critical for advancing microelectronics, superconductors, and catalysts. Ptychographic tomography enables non-destructive 3D analysis of nanostructures and hidden defects in microchips.
- In Geology and Industry: SR is applied to create "digital twins" of rock cores, study pore space in geological formations, and analyze fluid dynamics, thereby improving the efficiency of exploration and extraction of hard-to-recover oil and gas reserves.

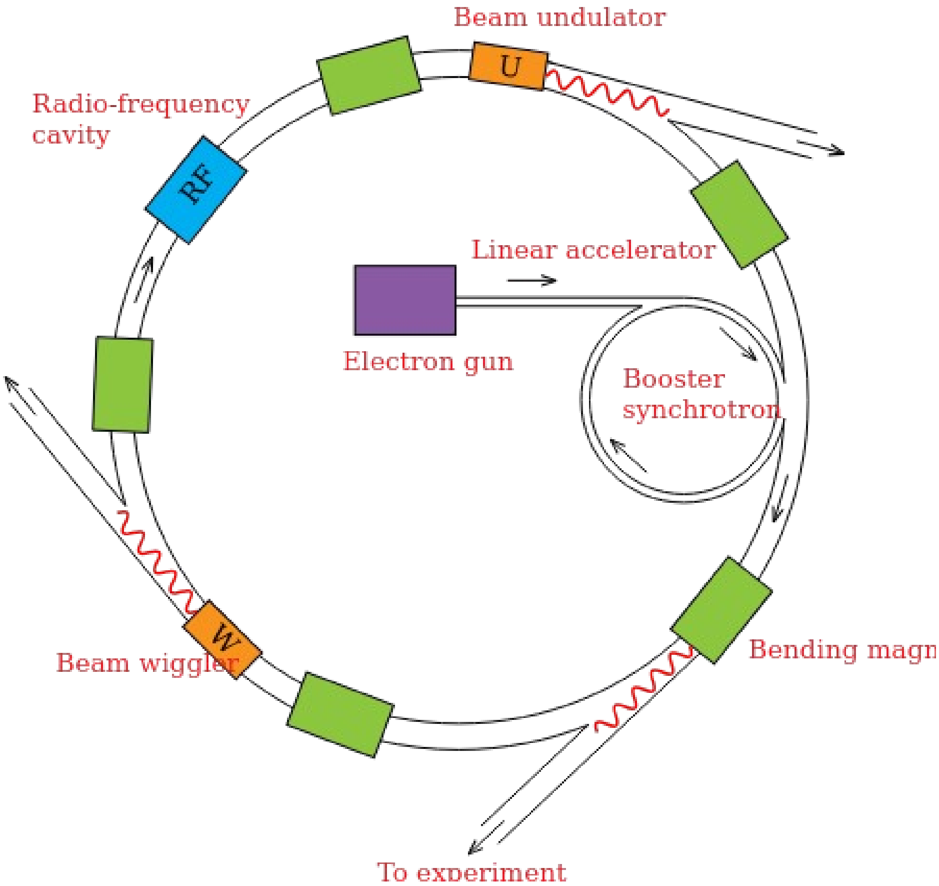

Figure 1 — schematic

One of the fundamental problems limiting beam quality and stability in a synchrotron is chromaticity — the dependence of the magnetic system's focusing properties on a particle's momentum (energy). Since a real particle beam has a spread in energies, particles with different momenta experience different focusing forces in the quadrupole magnets. This leads to a difference in their betatron oscillation frequencies (tune) and, consequently, to a range of negative effects: beam smearing (loss of phase coherence), emittance growth, excitation of nonlinear resonances, and, ultimately, particle loss. The problem is particularly acute in new-generation, fourth-generation facilities (such as MAX IV, Sirius, ESRF-EBS), where complex multi-bend achromat (MBA) lattice structures are used to achieve record-low emittance and ultra-high brightness SR, but which also amplify nonlinear and chromatic effects.

Analysis and Compensation of Chromaticity are central tasks in synchrotron design and operation. Traditionally, sextupole magnets are used to correct chromaticity by introducing a nonlinear dependence of focusing on momentum, compensating for the primary chromatic aberration. However, their adjustment is a complex optimization problem, as sextupoles also introduce nonlinear distortions that can destabilize the beam.

Modern research, as shown in the work [1] is focused on developing analytical and numerical methods for precise modeling of these effects. For instance, approaches involve defining a chromaticity index — a quantitative measure of the influence of resonances on electron dynamics. This allows for the optimization of linear lattice parameters (tune, dispersion) even before conducting computationally intensive particle tracking simulations. Minimizing this index for a synchrotron model with an emittance around 80 picometer-radians has been shown to provide a significant dynamic aperture (up to 5 mm) for particles with a momentum deviation of ±3%, confirming the method's effectiveness. Such techniques are becoming standard for designing magnetic lattices, ensuring a balance between the low emittance required for SR brightness and sufficient dynamic aperture to contain a beam with energy spread.

2. Theory

The dynamics of charged particle motion in accelerators characterizes their interactions with both accelerator components and other particles. Analysis of these dynamics enables optimization of operational parameters to meet design specifications. Such analysis necessitates data on current beam parameters, obtained through measurement or reconstruction techniques, alongside mathematical models describing charged particle motion.

Key concepts for describing motion dynamics include transverse and longitudinal oscillation frequencies, betatron functions, resonances, and chromaticity.

2.1. Betatron oscillations

Within the complex magnetic lattice of a particle accelerator, the trajectory of a charged particle is defined by a precise equilibrium orbit. However, any perturbation will disrupt this perfect equilibrium. Rather than being lost, the particle responds by undergoing stable, oscillatory motion about the reference orbit. These inherent oscillations are known as betatron oscillations. For the case of a particle circulating in a cyclic accelerator its linear betatron motion is not described by a simple harmonic oscillator. Instead, due to the periodic nature of the magnetic lattice that focuses and guides the beam, the dynamics are governed by a second-order differential equation with periodic coefficients. This fundamental equation of motion is Hill's equation, which is formulated within a co-moving coordinate system that follows the ideal particle along the design orbit [2]:

$$x''+K_x(z)x=0,\ y''+K_y(z)y=0, \tag{1}$$

where $x'=\frac{dx}{dz}, y'=\frac{dy}{dz}, K_x(z)=\frac{1}{B\rho}\frac{\partial B_x}{\partial x}+\frac{1}{\rho^2}, K_y(z)=\frac{1}{B\rho}\frac{\partial B_y}{\partial y}+\frac{1}{\rho^2}$, B – magtetic field, ρ – radius of curvature of the trajectory. Here, x corresponds to the horizontal plane, and y corresponds to the vertical plane.

The solution, for instance, in the vertical plane, can be found in the form of two independent solutions [4]:

$$y_1(z)=e^{\frac{i\phi z}{L}}p_1(z),\ y_2(z)=e^{\frac{i\phi z}{L}}p_2(z) \tag{2}$$

where ϕ is the accumulated horizontal ϕ_x or vertical ϕ_z betatron phase, which is determined by following relation:

$$\cos(\phi)=\frac{1}{2}trM(s) \quad (3)$$

$p_1(z)$ and $p_2(z)$ are periodic functions of z, and M(s) is the transition matrix [4]:

$$p_i(z+L)=p_i(z), i=1,2,$$

$$M(s)=M(s+\frac{L}{s})$$

where L is the lenght of the accelerator period.

The oscillatory motion of particles in the transverse phase space, characterized by their coordinates and conjugate momenta, exhibits a periodic nature; however, its temporal profile is non-sinusoidal. This specific characteristic necessitates a spectral description of the motion, which consequently reveals a composition of multiple harmonic components, derived from the fundamental betatron oscillation frequencies.

Within the context of a cyclic particle accelerator, the horizontal (Q_x) and vertical (Q_y) betatron tune values are defined as the number of complete betatron oscillations a particle undergoes per single revolution around the accelerator ring. This quantitative measure is formally expressed by the integral relation:

$$Q=\frac{1}{2\pi}\oint\frac{dz}{\beta(z)}, \quad (4)$$

where $\beta(z)$ represents the appropriate optics function—either $\beta_x(z)$ or $\beta_y(z)$—which is a periodic function of the longitudinal coordinate z around the accelerator's circumference C [2]. This function encapsulates the focusing properties of the magnetic lattice.

The precise values of the betatron tunes are counted among the most critical operational parameters of any accelerator facility. Their significance is paramount, as they directly govern the stability of the particle beam under the influence of nonlinear magnetic fields and inevitable imperfections. The position of the operational working point, defined by the pair (Q_x, Q_y), within the tune diagram—a plane populated by numerous resonance lines—fundamentally determines the beam's dynamic aperture and long-term stability. Consequently, these parameters exert a profound influence on key performance metrics, including the achieved luminosity in colliding-beam experiments and the spectral brightness in synchrotron radiation sources. It is for these reasons that the accurate measurement and subsequent adjustment of the betatron tunes constitute an essential and primary diagnostic undertaking during the commissioning phase of any new accelerator and during routine operation.

The integer portion of the betatron tune can be ascertained with relative simplicity by inducing a localized perturbation to the beam's orbit using a dedicated magnetic kicker or by analyzing the response to a known field error. The resulting coherent wave-like distortion of the closed orbit exhibits a number of full wavelengths per revolution exactly equal to the integer part of the tune.

The determination of the fractional part of the betatron tune requires a more refined spectral technique. It is typically measured by exciting the beam into coherent betatron oscillation (e.g., via a single kick) and then performing a spectral analysis, such as a Fast Fourier Transform (FFT), on a time series of beam position data. This data is acquired on a turn-by-turn basis from one or more beam position monitors (BPMs) located around the ring. The dominant frequency component in the resulting spectrum corresponds directly to the fractional part of the betatron tune.

2.2. Synchrotron Motion

Synchrotron motion refers to the oscillatory dynamics of charged particles along the longitudinal direction within an accelerator, under the influence of time-varying electromagnetic fields. A fundamental aspect of this motion is that it involves continuous particle acceleration.

To achieve efficient acceleration and attain high particle energies, it is essential to maintain a precise resonant condition between the particle’s orbital revolution frequency and the temporal variations of the guiding magnetic field. This requirement is addressed through a mechanism known as phase synchronization or phase stability. The underlying principle consists in ensuring that the accelerating electric field is applied in synchrony with the particle’s arrival at the radiofrequency (RF) cavity. As the magnetic field increases over time, the orbital circumference of high-energy particles also changes; thus, to preserve resonance, the frequency of the accelerating voltage or the strength of the magnetic field must be adjusted accordingly. When this condition is met, particles receive a net gain in energy per turn, allowing sustained acceleration over multiple revolutions until the target energy is reached.

The relation describing the dynamics of synchrotron motion:

$$\frac{p}{dp} = q \cdot B \cdot ds, \tag{5}$$

where q is the particle's charge, B is the local value of the magnetic induction varying along the orbit, and ds is the elementary path length traversed. This formula shows that the change in a particle's momentum is a function of its current momentum and the local strength of the inhomogeneous field.

The field inhomogeneity causes particles with different initial momentum values to receive unequal increments, leading to their dephasing relative to the equilibrium orbit. The resulting periodic oscillations are described by the equation of synchrotron oscillations, which in its simplest form resembles a harmonic oscillator equation:

$$\frac{d^2}{dt^2}\left(\frac{\Delta p}{p}\right)+\Omega_s^2\cdot\left(\frac{\Delta p}{p}\right)=0 \tag{6}$$

Here, $\Delta p/p$ is the deviation of the relative momentum from its equilibrium value, and Ω_s is the synchrotron oscillation frequency. The resulting momentum spread $\Delta p/p$ manifests as the amplitude of these steady-state oscillations.

2.3. Resonances

Resonances in circular accelerators represent a critical class of phenomena that arise when the frequency of particle oscillations coincides with characteristic frequencies of perturbing influences within the machine. These conditions can lead to resonant energy transfer, resulting in undesirable effects such as beam instability, uncontrolled emittance growth, and particle loss, thereby adversely impacting overall accelerator performance and compromising the quality of experimental results.

Resonant phenomena occur when the betatron tune Q, or a linear combination of its harmonics, approaches a rational number $\frac{n}{m}$) (where n and m are integers). Under such conditions, even small perturbative effects — such as magnetic field errors, misalignments, or nonlinearities — can be strongly amplified, leading to a degradation of beam quality.

For a more detailed examination of resonance mechanisms, it is instructive to consider the synchronous frequency ω_s in a synchrotron or booster. This frequency determines the condition under which charged particles remain in synchronism with the accelerating radiofrequency (RF) electric field. The synchronous frequency is a function of the particle's charge q, its rest mass m, the magnetic induction B_0 of the guiding field, and the accelerating RF frequency ω_{rf}. The maintenance of this synchronism is essential for stable longitudinal motion.

Longitudinal resonances occur when the frequency of synchrotron oscillations ω_s coincides with, or is an integer multiple of, the revolution frequency ω_{rf} of particles in the ring. Such a condition can disrupt the phase stability mechanism, leading to longitudinal instability, an increase in energy spread, and bunch dilution.

2.5 Beam Focusing System

By analogy with an optical system, a stable periodic structure composed of alternating focusing and defocusing magnetic elements can be created in a synchrotron. Similar to how a combination of converging and diverging lenses forms a stable light beam in optics, a specific sequence of magnetic sections — quadrupole lenses — provides transverse confinement and focusing of the charged particle beam, preventing its spreading by compensating for deviations. This principle, known as strong focusing, underlies all modern circular accelerators.

A key feature of magnetic focusing is the dependence of the focusing force on the particle's momentum. Ideal focusing is achieved for particles with the nominal, or equilibrium, momentum. However, a real beam always possesses a certain momentum spread, meaning it contains particles with deviations from the nominal value. As a result, particles with different momenta react differently to the same magnetic field: for some, an element acts as a focusing lens; for others, it is either too strong or too weak. This phenomenon is entirely analogous to chromatic aberration in optics, where lenses refract light of different wavelengths differently, leading to image blurring.

Thus, the momentum spread in the beam gives rise to the effect of chromatic aberrations in the synchrotron's magnetic lattice. It manifests as a dependence of transverse focusing on particle energy, which causes additional beam blurring, an increase in its emittance, and ultimately can lead to a loss of motion stability. This is precisely why combating chromatic aberrations — for example, by introducing special corrector elements or using sextupole magnets for chromatic correction — is a crucial engineering task in designing accelerator lattices, aimed at preserving a high degree of monochromaticity and a minimal beam size.

2.6 Chromaticity

Chromaticity represents a fundamental optical parameter in circular accelerators, characterizing the dependence of the transverse betatron oscillation frequencies on the particle's momentum deviation. This dependence arises from the variation in magnetic rigidity with particle energy, leading to differential focusing effects for particles within the beam's momentum spread. The presence of non-zero chromaticity can significantly influence the stability of particle motion and may result in resonant phenomena, emittance growth, and reduction of the accelerated beam's intensity. Consequently, precise measurement, control, and correction of chromaticity are essential procedures for ensuring the stable and efficient operation of accelerator systems.

The chromaticity ξ is formally defined as the derivative of the betatron tune Q with respect to the relative momentum deviation $\delta=\frac{\Delta p}{p_0}$:

$$\xi=\frac{\Delta Q}{\delta} \tag{7}$$

Thus, for a particle with a momentum deviation \(\delta\), the shift in its betatron tune from the nominal value Q_0 is given, to first order, by the linear relation $\Delta Q=\xi*\delta$. This linear approximation is valid for small momentum deviations and provides the foundational basis for chromaticity measurement and correction techniques.

A standard method for determining the chromaticity involves introducing a controlled change to the beam's revolution frequency. By applying a small frequency shift Δf to the radiofrequency (RF) system's master oscillator, the synchronous condition is altered, resulting in a change in the beam's average momentum. The resultant relative momentum shift is given by:

$$\frac{\Delta p}{p}=\frac{-1}{\eta}\frac{\Delta f}{f_0} \tag{8}$$

where f_0 is the nominal revolution frequency, and η is the frequency slip factor, defined as $\eta=\alpha-\frac{1}{\gamma^2}$. Here, α is the momentum compaction factor, characterizing the dependence of the orbit length on momentum, and γ is the relativistic Lorentz factor.

By measuring the corresponding shift in the betatron tune ΔQ resulting from this induced momentum change, the linear chromaticity can be directly calculated using the expression:

$$\xi=\frac{-\eta \Delta Q f_0}{\Delta f} \tag{9}$$

This experimental method provides a direct measurement of the global chromaticity of the ring.

From a theoretical perspective, the natural chromaticity of a lattice is determined by the integrated focusing properties of its magnetic elements. In the case of linear magnetic optics, the chromaticity is derived from the actions of the quadrupole lenses and can be expressed by the integral:

$$\xi=-\frac{1}{4\pi}\oint K(z)\beta(z)dz, \tag{10}$$

where $K(z)$ is the quadrupole gradient function, and $\beta(z)$ is the corresponding beta function at the longitudinal position z. This integral is taken over the complete circumference of the accelerator. The negative sign indicates that for a standard focusing lattice, the natural chromaticity is negative, meaning the betatron tune decreases for particles with higher momentum. To ensure stability against head-tail instabilities and other chromatic effects, this natural chromaticity is typically corrected towards zero or slightly positive values using dedicated sextupole magnets, which introduce a nonlinear momentum-dependent focusing term to compensate the inherent chromatic aberration of the lattice. The betatron tune shift of a particle resulting from chromaticity is expressed as [6]:

$$\Delta v(N)=\xi\delta cos(2\pi vs\ N+\phi s\ ), \tag{11}$$

where ξ represents the chromaticity, N is the time measured in turns, δ is the synchrotron amplitude, and ϕ_s is the synchrotron phase at $N=0$. The synchrotron amplitude is given in relative energy units, meaning the actual maximum energy displacement of the particle equals δ multiplied by the nominal energy E_0. Denoting σs as the rms relative energy spread, the actual rms energy spread becomes $\sigma_s\, E_0$. Consequently, the betatron phase shift is:

$$\Delta\phi(\delta, \phi s\ , N)=2\pi \int 0 N\ dN'\,\Delta v(N')=D\delta, \tag{12}$$

with the coefficient D defined by:

$$D=2\xi v_s^{-1}\sin(\pi v_s\ N)\cos(\pi v_s\ N+\phi_s) \tag{13}$$

The initial step involves determining the distribution of particles in betatron phase as the beam undergoes decoherence. This distribution starts as a delta function at $N=0$ and broadens over time. However, after a complete synchrotron cycle, it reverts to a delta function because the sinusoidal tune shifts average to zero over a full period. The distribution is derived by executing a change of variables within the synchrotron phase space distribution. A practical approach is to represent the betatron distribution as an integral over a Dirac delta function, leading to:

$$\rho(\phi, N)=\int_0^{\infty} d\delta \int_0^{2\pi} d\phi_s\ \rho_s(\delta)\delta(\phi-\Delta\phi(\delta, \phi_s\ , N)), \tag{14}$$

where synchrotron phase space distribution is described by:

$$\rho_s(\delta)=\frac{1}{2\pi\sigma_s^2}\ \delta e^{-\delta^2/2\sigma_s^2} \tag{15}$$

Performing the integration over δ yields:

$$\rho(\phi, N)=\frac{1}{2}\int_0^{2\pi} d\phi s\ D^{-1}\rho_s(\phi/D) \tag{16}$$

where D maintains its previous definition. The integration over all phases ϕs results in double counting of the Dirac delta function peaks, which is compensated by the factor of $1/2$. This integral is evaluated through the change of variables $u=\phi tan(\pi v_s\ N+\phi_s)$.

This substitution leads to:

$$\rho(\phi, N)=\frac{\phi}{\pi\alpha^2}\ e^{-\phi^2/2\alpha^2}\int_0^{\infty} du e^{-\phi^2u^2/2\alpha^2}=\frac{1}{\sqrt{2\pi}\,|\alpha|}\ e^{-\phi^2/2\alpha^2} \tag{17}$$

where the parameter α is defined as $\alpha=2\sigma_s\ \xi v_s^{-1}\sin\pi v_s\ N$. Assuming the particles were initialized with betatron amplitude a, the centroid of the distribution ρ exhibits an amplitude $\bar{a}(N)=aA_s(N)$, with the decoherence factor A_s given by:

$$A_s(N)=\int_{-\infty}^{\infty} d\phi cos\phi \rho(\phi,N)=e^{-\alpha^2/2} \tag{18}$$

where α is as defined above. The integration limits extend to infinity to incorporate particles that have "lapped" those with $\phi=0$.

For the coherently oscillating beam centroid on the n^{th} revolution turn is given by:

$$x(n)=x_0 F_\delta \cos(2\pi \nu_\beta n+\phi_\beta), \tag{19}$$

where x_0 represents the initial amplitude of the betatron oscillation induced by the kick, ν_β is the nominal betatron tune, and ϕ_β is the initial phase of the oscillation.

The term F_δ is a modulation envelope function that governs the decoherence of the coherent betatron motion due to the finite momentum spread σ_δ within the beam and the lattice's chromaticity ξ. For an ensemble of particles characterized by a Gaussian momentum distribution, this envelope function takes the form of a Gaussian damping factor:

$$F_\delta=\exp\left(-\frac{a^2}{2}\right), \qquad a=\frac{2\xi\sigma_\delta \sin(\pi\nu_s n)}{\nu_s} \tag{20}$$

Here, ν_s denotes the synchrotron tune, which defines the frequency of longitudinal synchrotron oscillations. The function $\alpha(n)$ encapsulates the phase advance spread accumulated over n turns, which arises from the chromatic tune spread $\xi\sigma_\delta$. The sinusoidal dependence on the synchrotron tune ν_s reflects the periodic bunching and debunching of particles in longitudinal phase space, which modulates the effective tune spread experienced by the beam centroid.

Equation (20) indicates that the centroid of the beam distribution undergoes coherent betatron oscillations whose amplitude is modulated by the envelope F_δ. This modulation occurs with a frequency inversely proportional to the synchrotron tune, ν_s, meaning the envelope repeats every $\frac{1}{\nu_s}$ turns. It is crucial to note that this amplitude modulation affects only the coherence of the oscillating wave packet but does not alter the frequency of the oscillation itself; therefore, the measurement of the betatron tune ν_β remains unaffected.

The Fourier spectrum of the time-domain signal described by Equation (20) is characterized by a central peak at the frequency ν_β, corresponding to the coherent betatron oscillation. The decoherence process, governed by the Gaussian envelope F_δ imparts a specific spectral shape to this peak. Consequently, the resulting line shape in the frequency domain is also Gaussian, with a spectral width that is directly proportional to the product of the chromaticity and the momentum spread, $\xi\sigma_\delta$ [7].

3. The practic

3.1. Aim

The aim of this work is to develop an algorithm for the quantitative assessment of the chromatism of betatron oscillations in a circular accelerator and to conduct a comparative analysis of its magnitude for two states of the magnetic lattice: with an active and an inactive sextupole corrector. Within the scope of this task, it is required to identify mathematical indicators that allow for the detection of changes in the chromatic properties of the system and to establish the dependence of the degree of chromaticity compensation on the operation of the sextupole.

3.2 Choice of Programming Language

The selection of a programming language for signal analysis is driven not only by convenience and workflow efficiency but also by the accessibility of a comprehensive toolkit for scientific computation and data processing. For this purpose, the language Python, version 3.7 or newer, was chosen.

Python offers several key advantages that make it well-suited for signal analysis:

1. The SciPy library supplies an extensive set of functions for scientific and technical computations, encompassing signal processing, optimization, interpolation, and numerous other data analysis operations.
2. Data analysis libraries such as NumPy and Pandas provide capabilities for the efficient storage, manipulation, and analysis of large datasets.
3. Visualization libraries like Matplotlib Seaborn and Plotly facilitate the generation of scientific graphs with high-speed rendering.
4. The language's modularity and extensibility simplify the integration of specialized third-party libraries and modules tailored for signal analysis.
5. A large and active community ensures access to a vast pool of knowledge, resources, and tools for addressing a wide array of tasks.

The choice of Python version 3.11 or higher is justified by the fact that contemporary libraries and tools most fully support the latest versions of the language, thereby guaranteeing optimal performance and functionality for signal analysis tasks.

3.3. Data

Two sets of data files were used for the analysis, each containing 24 files. One set contained information about betatron oscillations with the sextupole turned off, and the other set with the sextupole turned on. These files consist of time series data obtained during accelerator operation. Each of these files contained information about oscillations in two planes: X and Y.

3.4. Data processing

The data underwent preprocessing — filtering of heavily noisy data, data with outliers, and sharp amplitude spikes. Such data were discarded during the preprocessing stage. An example of such data is shown in Figure 2.

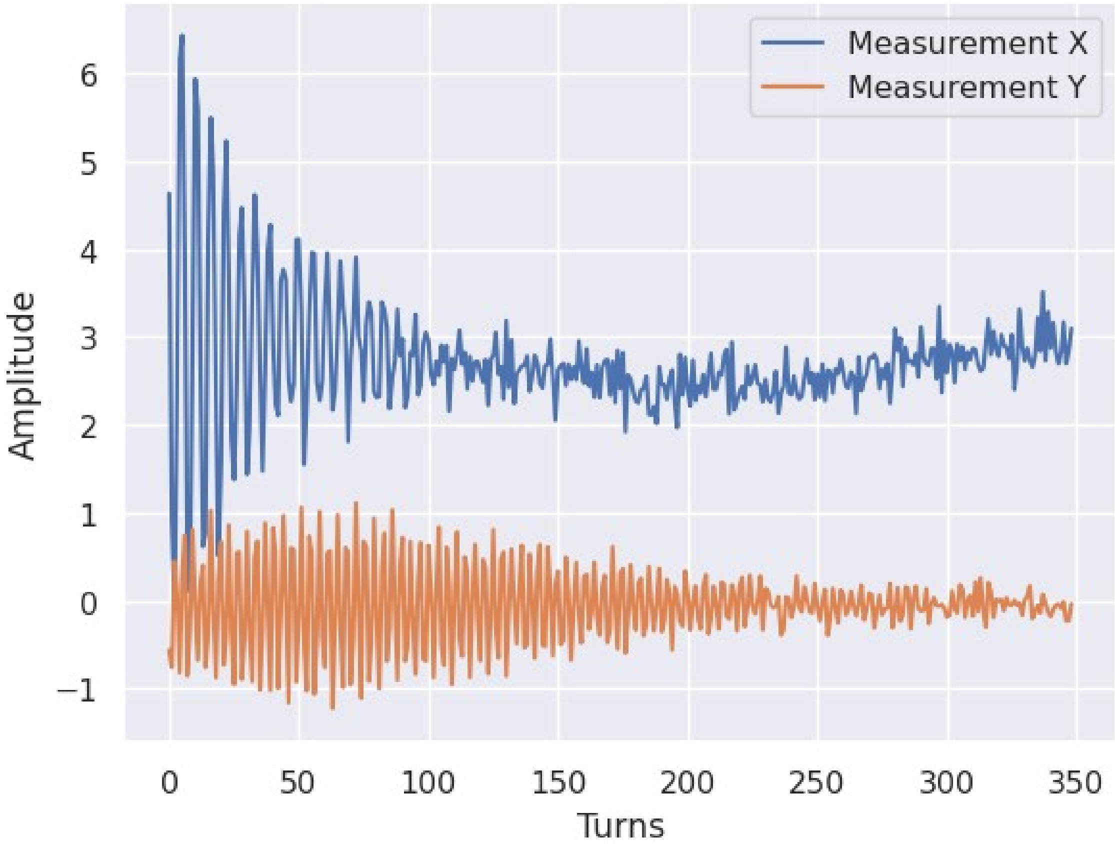

Figure 2 - Experimental data

Further preprocessing included centering the oscillations around zero and normalizing them to the maximum absolute value of the function. Centering was performed by subtracting the mean value of the function.

3.5. Determination of Chromaticity

To construct an approximating function, it is necessary to have some initial parameters from which the approximation algorithm will proceed. The oscillation frequency value was chosen as the main initial parameter. Frequency analysis was performed using discrete Fourier transform [3].

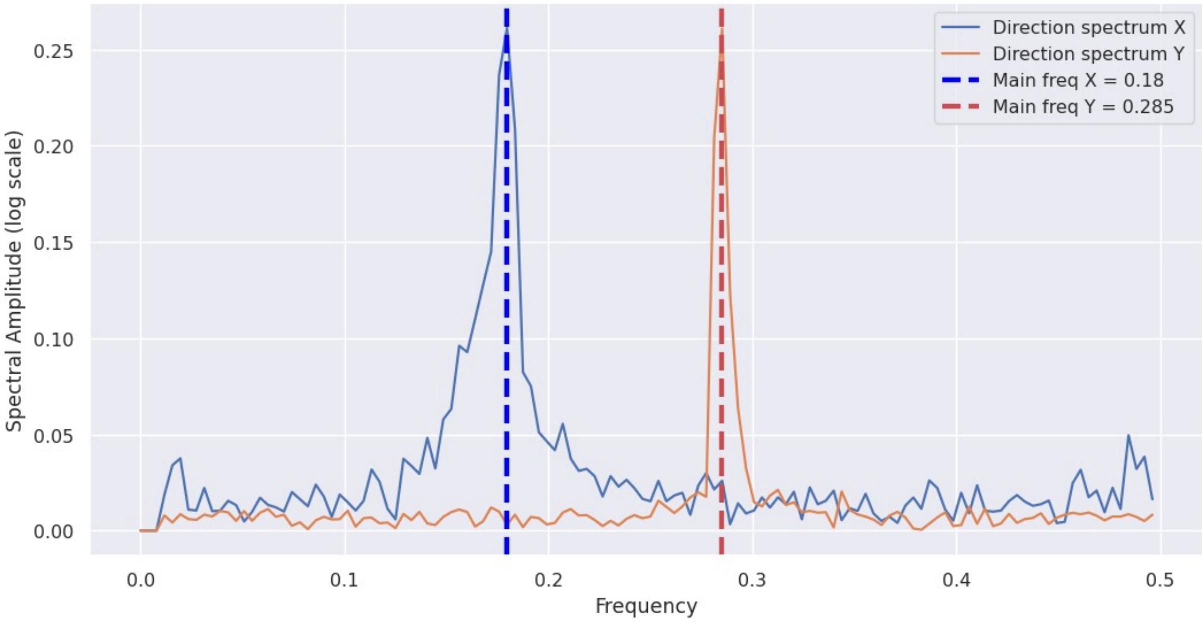

Figure 3 - Frequency analysis

For approximation, function (20) was used. The obtained frequencies were subsequently used in the approximation process, with the frequency specified not as a fixed value but within an interval. The initial phase was defined within a range. The amplitude parameters were set in the range — due to preprocessing (normalization), this range is universal for all oscillations in the dataset. Centering eliminates the need to introduce an additional term for shifting the approximation function.

Approximation was performed not for all data but only for the segment where damped oscillations were observed. Experimentally, it was determined that the optimal number of points for approximation is the first 50. Figure 4 shows a graph of the approximation of damped oscillations.

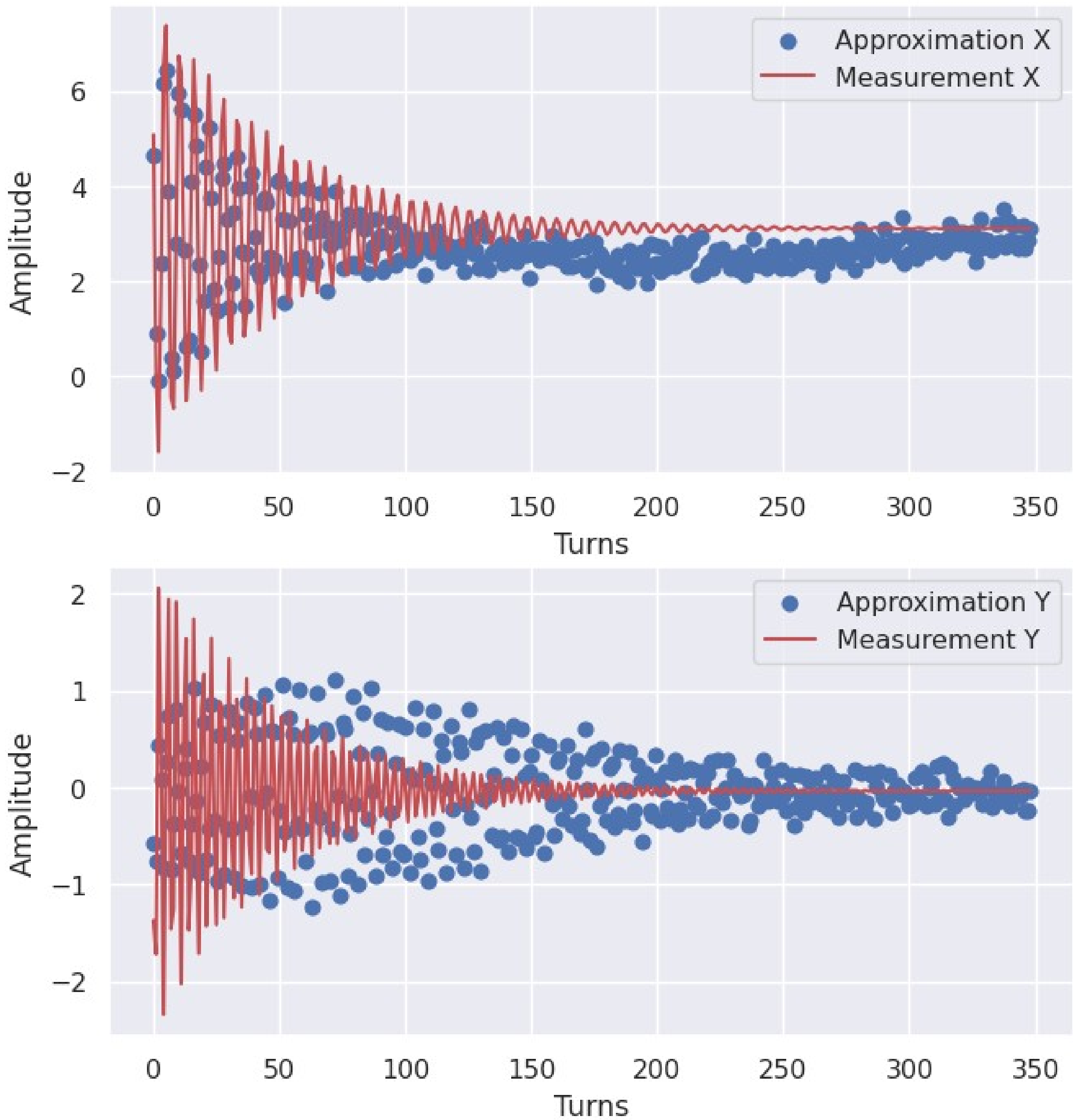

Figure 4 – Approximation of damped oscillations

The approximation was performed for data with the sextupole turned on and turned off. As a result of the approximation, the chromaticity values averaged over all spectra were obtained. The obtained average values of the first-order chromaticity for oscillations with the sextupole turned on and turned off, for oscillations in the X-plane and Y-plane, are presented in Table 1.

Table 1 — Average chromaticity values with the sextupole turned on and turned off

sextupole turned off		sextupole turned on	
X	Y	X	Y
3.825 ± 0.28	3.781 ± 0.36	5.4999 ± 0.16	5.4871 ± 0.21

Conclusion

This work investigated the betatron oscillations of particles in an accelerator complex and conducted their detailed analysis. To achieve this goal, a mathematical approximation was constructed that accurately describes the behavior of the betatron oscillations.

The central part of the research was the study of chromaticity in the betatron oscillations. The analysis was performed for two key accelerator operational modes: with the sextupole corrector turned on and off. The conducted comparison revealed significant differences in the nature and magnitude of chromaticity between these two configurations, confirming the substantial influence of the sextupole on the dispersive properties of the system.

As part of the work, specialized algorithms for data processing and oscillation analysis were also developed and applied, which enabled the acquisition of highly accurate and reliable results.

As an outcome of the work, it was established that controlling the state of the sextupole is an effective tool for controlling and correcting the chromatic aberrations of the beam. Software was developed to perform the necessary calculations and visualization.

For the further development of this research, promising directions include: refining the mathematical model to account for non-linear effects, conducting a more in-depth study of the dependence of chromaticity on oscillation amplitude, and adapting the obtained results for the tasks of optimizing the accelerator's working point.

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