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Interaction energy of dinuclear and trinuclear systems

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CONTENTS

ABSTRACT

The report presents a theoretical study on the interaction energy of dinuclear and trinuclear systems composed of heavy nuclei, focusing on their formation and energy characteristics. The calculation of interaction energy involves nuclear deformations, Coulomb interactions, and nuclear forces. Dinuclear systems (DNS) are formed during the low-energy nuclear collisions, where a balance between attractive nuclear forces and Coulomb repulsion plays a key role. Trinuclear systems (TNS) can be formed during the spontaneous ternary fission of heavy nuclei. The report outlines the mathematical formulations used to describe interaction energy of these systems for the spontaneous ternary fission of ^{252}Cf as an example case. Numerical results illustrate the potential energy surfaces of these systems under different configurations, providing a comparative analysis of the resulting energy values.

INTRODUCTION

Theoretical description of the interaction energy of nuclear systems is a crucial aspect of modern nuclear physics, particularly when studying complex structures such as dinuclear and trinuclear systems. These systems consist of multiple heavy nuclei interacting with each other, and theoretical investigation of their interaction energy is important in understanding the fundamental mechanisms of nuclear interactions, as well as in developing new models to describe nuclear reactions [1].

The formation of dinuclear and trinuclear systems involving heavy nuclei is governed by a complex interplay of forces, including strong nuclear interactions, Coulomb repulsion, and centrifugal forces. In dinuclear systems, such as clusters composed of two heavy nuclei, the key factor is the balance between the short-range attraction due to strong nuclear forces and the long-range Coulomb repulsion. These forces together form a potential barrier and determine possible bound states within the system .

Dinuclear systems (DNS) involving heavy nuclei are formed during low-energy collisions between two nuclei, after the capture of projectile nucleus by target. Interaction energy of two nuclei, in dependence on relative distance coordinate R between centers of nuclei, is schematically shown in Fig. 1. When the nuclei overcome the Coulomb barrier, due to dissipation of kinetic energy, two nuclei can be trapped in the potential pocket and they may form a quasi-stationary system where the resulting forces keep them at a certain distance from each other. The minimum of interaction energy is located approximately at a distances $R_{min}=R_1+R_2+0.5$fm. Due to the nucleon exchange between DNS nuclei, system can evolve to different asymmetries, including the formation of compound nucleus [2].

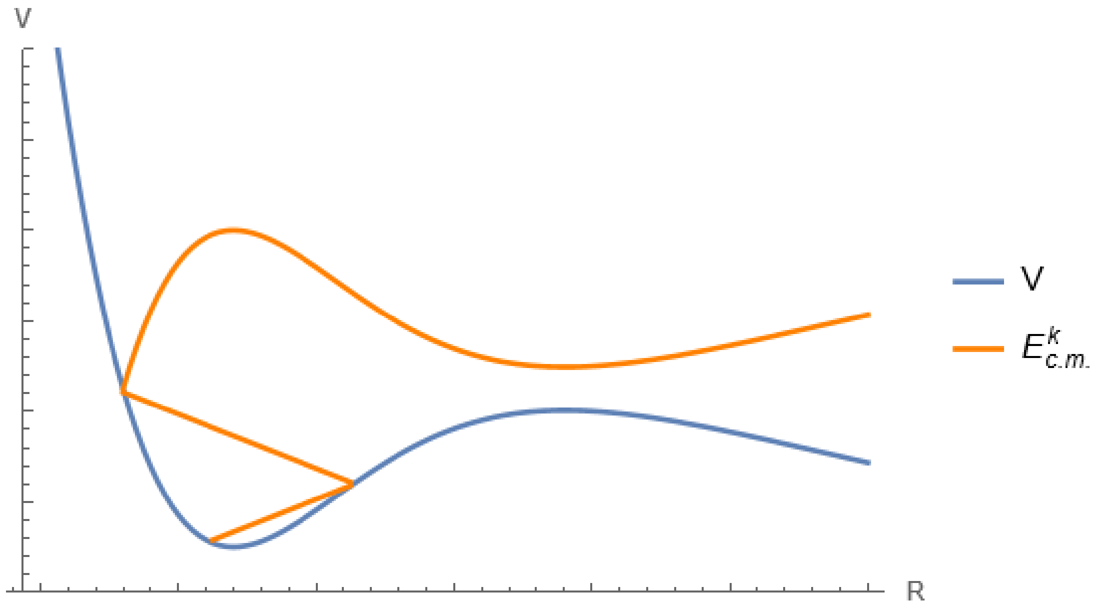

Figure 1. DNS formation during low energy nuclear collisions.

Trinuclear systems (TNS) are formed as a result of the spontaneous nucleon redistribution process during the spontaneous ternary fission of heavy nuclei. For example, in spontaneous ternary fission of ^{252}Cf, the clusterization process leads to the formation of asymmetric DNS. Further, the nucleon exchange process leads to the formation of symmetric dinuclear systems. Due to the presence of excitation energy, in the surface of one of the nuclei of DNS, the light cluster can be formed, thus leading to the formation of a system composed of three nuclei. The interactions between all three nuclei govern its subsequent evolution. The formation of a TNS involves a redistribution of energy and interaction forces, requiring careful investigation in theoretical models, particularly when describing the potential energy and stability of such systems.

1. Interaction energies of dinuclear and trinuclear systems

Below we will consider the formation of DNS and TNS during spontaneous ternary fission of ^{252}Cf. In this case the centrifugal forces do not contribute. The nuclei in DNS assumed to have axial symmetry. The configuration of nuclei in the DNS formed during the spontaneous decay of ^{252}Cf is depicted in Figure 2.

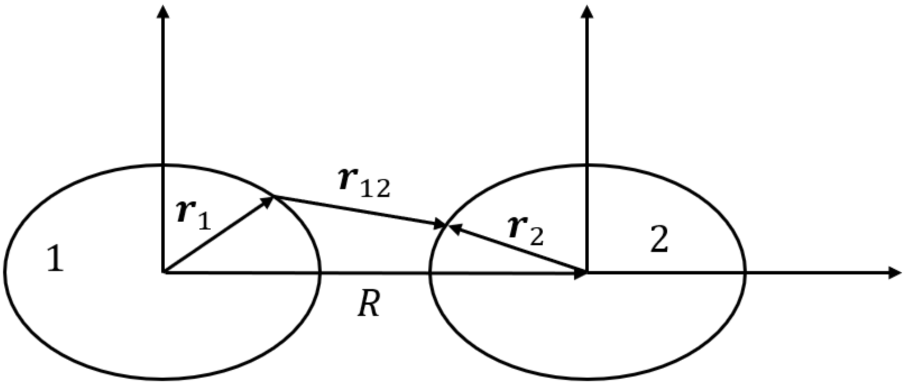

Figure 2.

Due to the Coulomb forces, dinuclear system has minimal energy in tip-to-tip configuration as illustrated in Fig. 2. The interaction energy of a dinuclear system is given by the following formula:

$$V_{\rm DNS}(R) = V_{Coul}(R) + V_N(R), \tag{1}$$

The interaction energy of the corresponding trinuclear system is given by combination of two body interactions:

$$V_{\rm TNS}(R) = V_{Coul}^{12}(R) + V_N^{12}(R) + V_{Coul}^{23}(R) + V_N^{23}(R) + V_{Coul}^{13}(R), \tag{2}$$

The configuration of the TNS is shown in Figure 3.

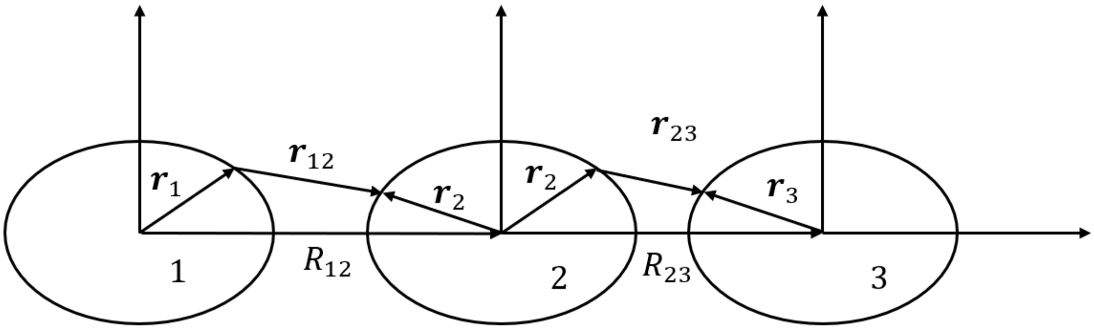

Figure 3.

We neglect the nuclear interaction between nuclei 1 and 3 due to the large distance between them.

2. The coulomb interaction in dinuclear system

The coulomb interaction is given by well-known formula:

$$V_{Coul}(R) = e^2 \int\int \frac{\rho_1^z(\boldsymbol{r}_1)\rho_2^z(\boldsymbol{r}_2)}{\boldsymbol{r}_{12}} d\boldsymbol{r}_1 d\boldsymbol{r}_2 \,, \tag{3}$$

We consider the charge density distribution in both nuclei as:

$$\rho_i^z(\boldsymbol{r}_i) = \rho_{0,i}^z \theta(R_i(\vartheta, \varphi, \Theta) - r), \tag{4}$$

where $\theta(x)$ is the step function and

$$R_i(\vartheta, \varphi, \Theta) = r_0 A_i^{\frac{1}{3}} \left(1 + \beta_2^i Y_{20}(\vartheta_{i0})\right) =$$
$$= r_0 A_i^{\frac{1}{3}} \left(1 + \beta_2^i \sqrt{\frac{4\pi}{5}} \sum_m (-1)^m \, Y_{20}(\vartheta, \varphi) Y_{20}(\Theta, \Phi) \right), \tag{5}$$

is the distance of the deformed nuclear surface from the center in an intrinsic coordinate system of the nucleus *i* [3, 4]

To calculate integral (3), we will use the following decomposition of r_{12}, considering the scheme in Figure 2:

$$\frac{1}{r_{12}} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{1}{2l+1} r_1^l Y_{lm}(\vartheta_1, \varphi_1) \times$$

$$\times \sqrt{\frac{1}{(2l)!}} \sum_{\substack{l_1,l_2=0 \\ l_2-l_1=l}} (-1)^{l_1+l_2} \sqrt{\frac{(2l_2+1)!}{(2l_1+1)!}} C_{l_1 m l_2 0}^{lm} \frac{r_2^{l_1}}{R^{l_2+1}} Y_{lm}(\vartheta_2, \varphi_2)\,, \tag{6}$$

then, potential energy of Coulomb interaction is:

$$V_{Coul}(R) = e^2 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{1}{2l+1} \int r_1^l \rho_1^z(r_1) Y_{lm}(\vartheta_1, \varphi_1)\, dr_1 \times$$

$$\times \sqrt{\frac{1}{(2l)!}} \sum_{\substack{l_1,l_2=0 \\ l_2-l_1=l}} (-1)^{l_1+l_2} \sqrt{\frac{(2l_2+1)!}{(2l_1+1)!}} C_{l_1 m l_2 0}^{lm} \frac{1}{R^{l_2+1}} \int r_2^{l_1} \rho_2^z(r_2) Y_{lm}(\vartheta_2, \varphi_2) dr_2\,, \tag{7}$$

The integrals in (7) with parameters (*l = 0, m = 0*) is:

$$\int r_i^l \rho_i^z(r_i) Y_{lm}(\vartheta_i, \varphi_i)\, dr_i = \frac{Z_i}{\sqrt{4\pi}}, \tag{8.1}$$

With parameters (*l=2, m=0*) is:

$$\int r_i^l \rho_i^z(r_i) Y_{lm}(\vartheta_i, \varphi_i)\, dr_i = Z_i \sqrt{\frac{4\pi}{5}} \frac{3}{4\pi} R_{0,i}^2 \beta_i Y_{lm}(\Theta_i) + Z_i \frac{3}{7\pi} \left(R_{0,i} \beta_i\right)^2 Y_{lm}(\Theta_i), \tag{8.2}$$

After integration we obtain:

$$V_{Coul}(R) = \frac{e^2 Z_1 Z_2}{R} + \frac{3}{5}\frac{e^2 Z_1 Z_2}{R^3}\sum_{i=1,2} R_{0,i}^2 \beta_i Y_{lm}(\Theta_i) +$$

$$+ \frac{12\sqrt{5}}{35\sqrt{4\pi}}\frac{e^2 Z_1 Z_2}{R^3}\sum_{i=i,2}\left(R_{0,i}\beta_i\right)^2 Y_{lm}(\Theta_i)\,, \tag{9}$$

In tip-to-tip orientation, $\Theta_1 = \Theta_2 = 0$ and Coulomb interaction is minimal between DNS nuclei.

3. The nucleon-nucleon interaction in dinuclear system

The nuclear interaction in dinuclear system we calculate by double folding formalism [2]:

$$V_N(R) = \int \rho_1(r_1)\rho_2(R - r_2)F(r_1 - r_2)\, dr_1 dr_2, \tag{10}$$

where nucleon-nucleon forces are taken as

$$F(r_1 - r_2) = C_0\left(F_{in}\frac{\rho_0(r_1)}{\rho_{00}} + F_{ex}\left(1 - \frac{\rho_0(r_1)}{\rho_{00}}\right)\right)\delta(r_1 - r_2),$$

$$F_{in,ex} = \left(f_{in,ex} + f'_{,ex}\tau_1\tau_2\right) + \left(g_{in,ex} + g'_{in,ex}\tau_1\tau_2\right)\sigma_1\sigma_2, \tag{11}$$

There τ and σ are isospin and spin matrices, respectively. Parameters C_0, f, f', g, g' are determined from the description of a large set of experimental data on the theory of finite Fermi systems [5]. If we neglect the spin dependence of the nucleon-nucleon interaction, we obtain:

$$V_N(R) = C_0\left\{\frac{F_{in} - F_{ex}}{\rho_{00}}\left(\int \rho_1^2(r)\rho_2(R - r)dr + \int \rho_1(r)\rho_2^2\,(R - r)dr\right) + F_{ex}\int \rho_1^2(r)\rho_2(R - r)dr\right\},$$

$$F_{in,ex} = f_{in,ex} + f'_{in,ex}\frac{N_1 - Z_1}{A_1}\frac{N_2 - Z_2}{A_2}, \tag{12}$$

where $C_0 = 300MeV, f_{in} = 0.09, f'_{ex} = 0.42, f_{ex} = -2.59, f'_{ex} = 0.54$.

For heavy nuclei, we can use the Fermi distribution of nucleon density

$$\rho_i(r) = \frac{\rho_{00}}{1 + exp\left(\frac{r - R_i(\vartheta'_i, \varphi'_i)}{a_0}\right)}, \qquad (13)$$

Where $R_i(\vartheta'_i, \varphi'_i)$ is the radius of deformed nucleus as in (5).

4. Numerical results

For numerical calculations, we used the following parameter values:

$$\rho_{00} = 0.17fm^{-1},$$

$$r_0 = 1.16fm,$$

$$a_0 = 0.56fm,$$

The values of the quadrupole deformation coefficients β of (2^+)-states were taken from the experimental data in tables of Raman [6]. The model was applied to the calculation of various interaction energy between DNS and TNS nuclei formed during the spontaneous ternary fission of ^{252}Cf. In calculations, the nucleus 3 is fixed to be ^{132}Sn due to the fact that most energetically favorable DNS configuration is ^{120}Cd+^{132}Sn and Sn is a magic nucleus. Then the nucleus ^{120}Cd can evolve to form different DNS configurations.

Figure 4 shows interaction energies of dinuclear subsystems ^{70}Ni+^{50}Ca and ^{50}Ca+^{132}Sn.

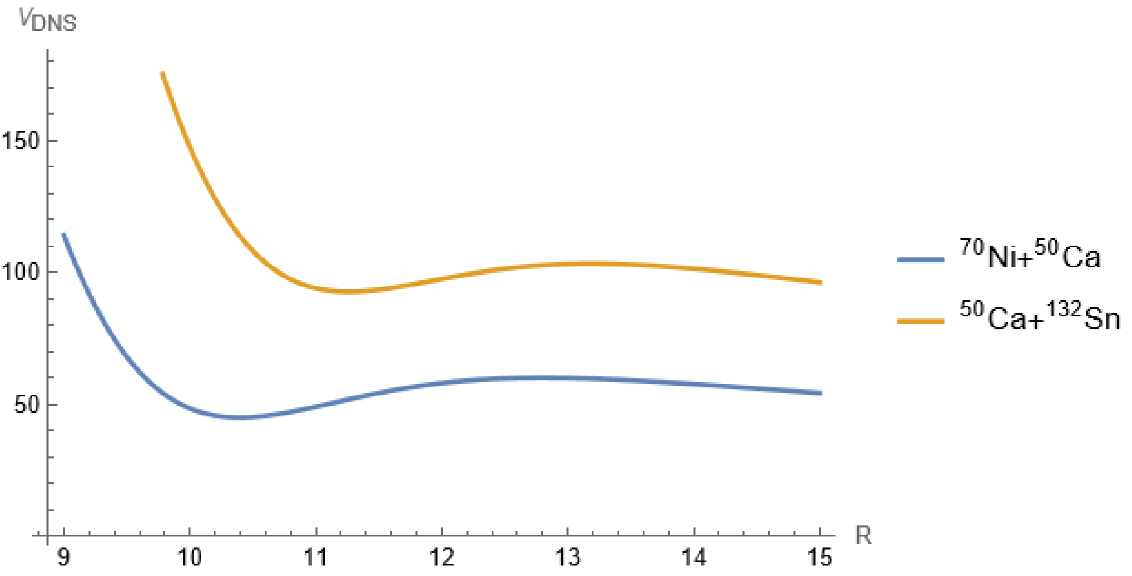

Figure 4.

The decay barrier B_R of DNS ^{70}Ni+^{50}Ca (^{50}Ca+^{132}Sn) in the absence of third nucleus is 15.054 MeV (10.63 MeV).

Calculated interaction energy of trinuclear system ^{70}Ni+^{50}Ca+^{132}Sn is shown on Figure 5. *R12* and *R23* are distances between mass centers of nuclei in ^{70}Ni+^{50}Ca and ^{50}Ca+^{132}Sn respectively. The global minimum of potential energy is marked by a blue dot $V_{min} = 230.786 MeV$, while the potential barriers of the TNS are indicated by red and orange lines, respectively.

TNS is located at the minimum of interaction potential at distances $R_{12} = 10.6 fm$, $R_{23} = 11.4\ fm$. The barrier heights are $B_{R12} = 6.88\ MeV, B_{R23} = 4.15\ MeV$. One can see that due to the Coulomb forces between TNS nuclei the DNS decay barriers are strongly decreased. Figures 6 and 7 show evolution of decay barriers B_{Ri} when third fragment is moving away. At large distances between DNS nuclei and third fragment the decay barriers B_{Ri} coincide with those values for DNS case.

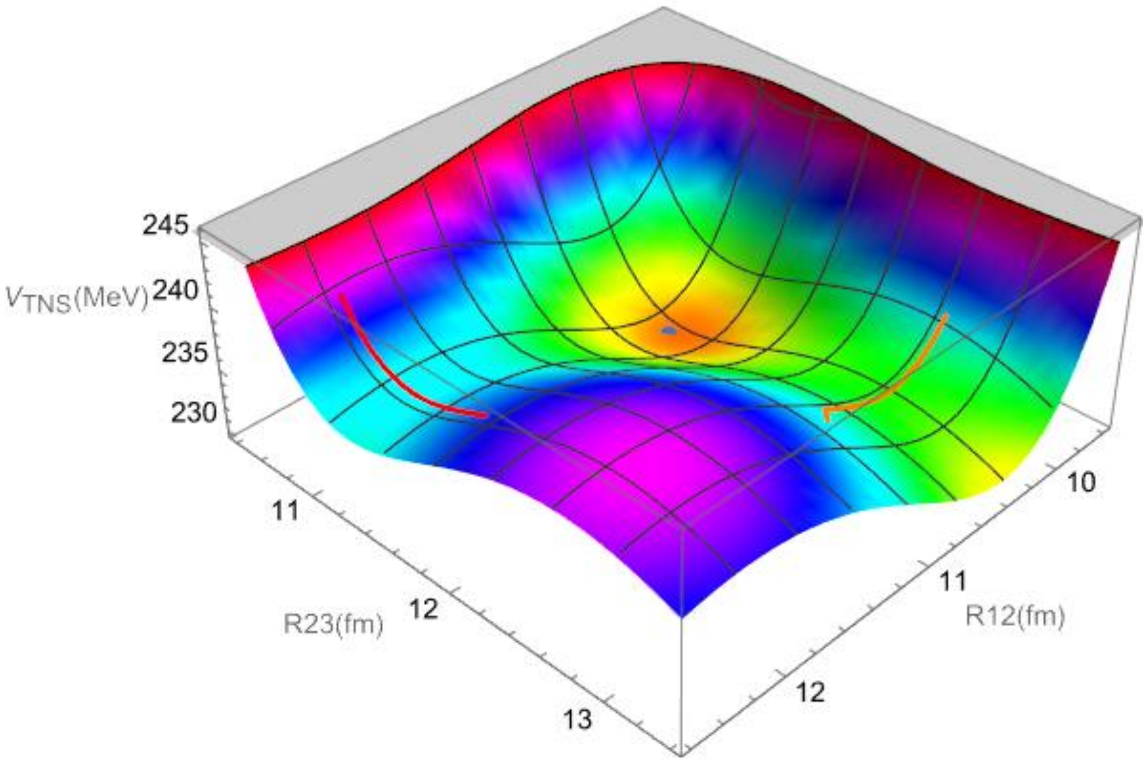

Figure 5.

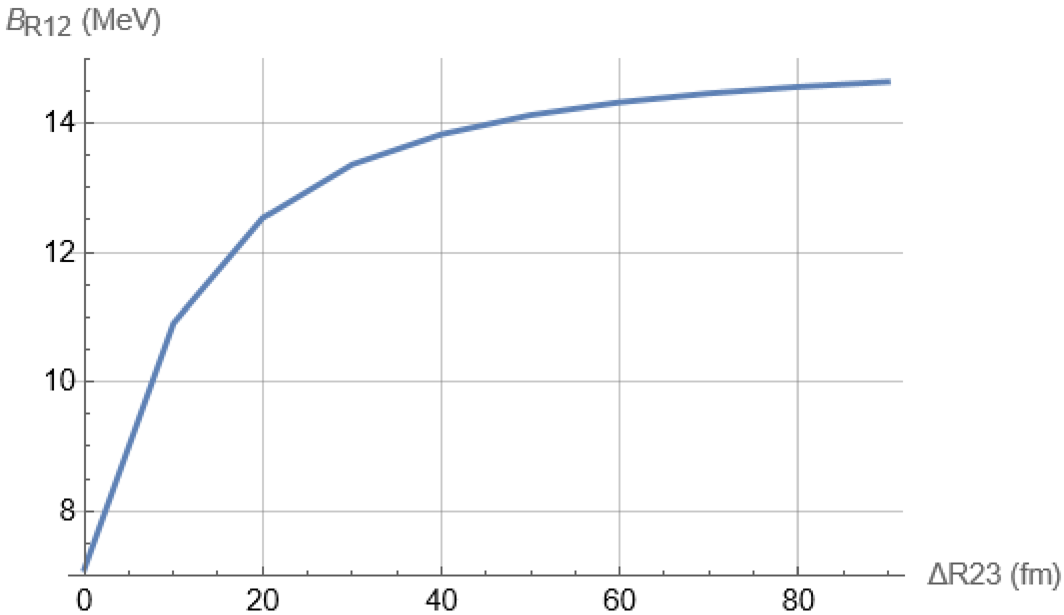

Figure 6.

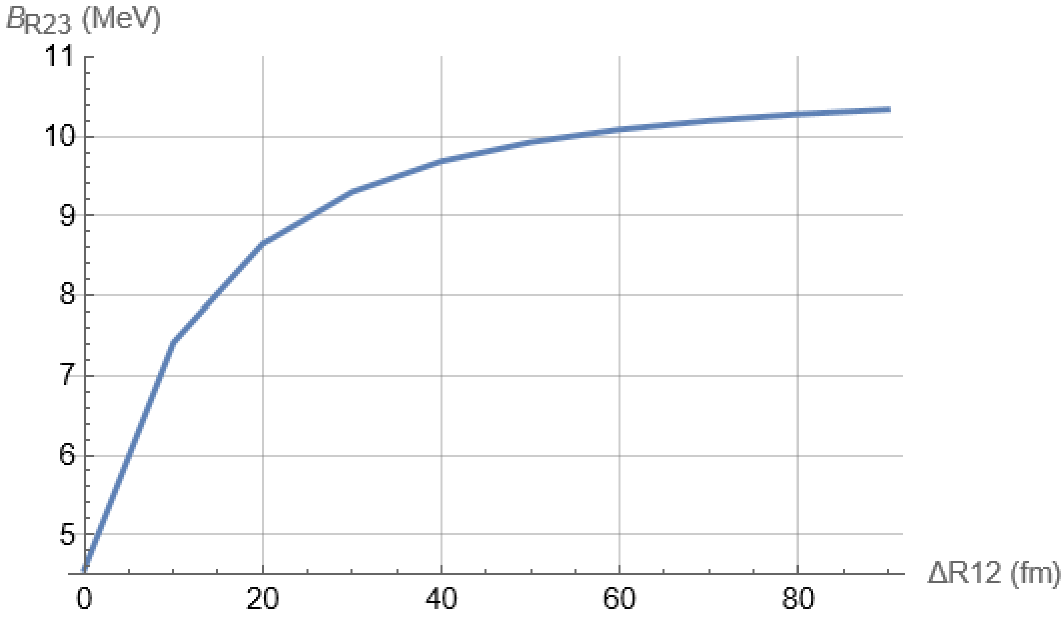

Figure 7.

Figure 8 and 9 shows the interaction potentials for DNS and TNS in the case when Ca and Ni nuclei are replaced by each other.

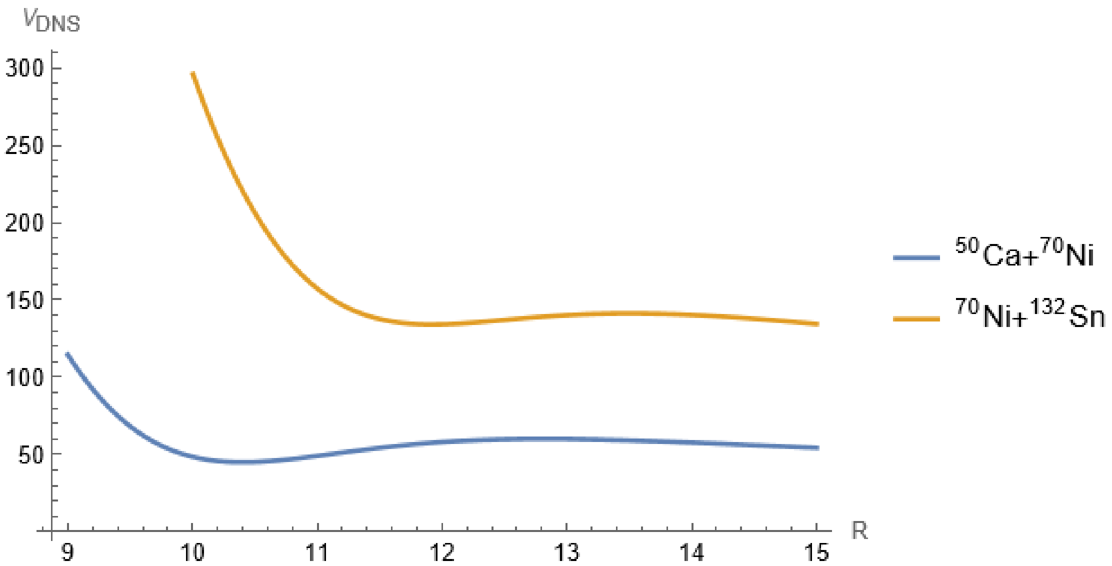

Figure 8.

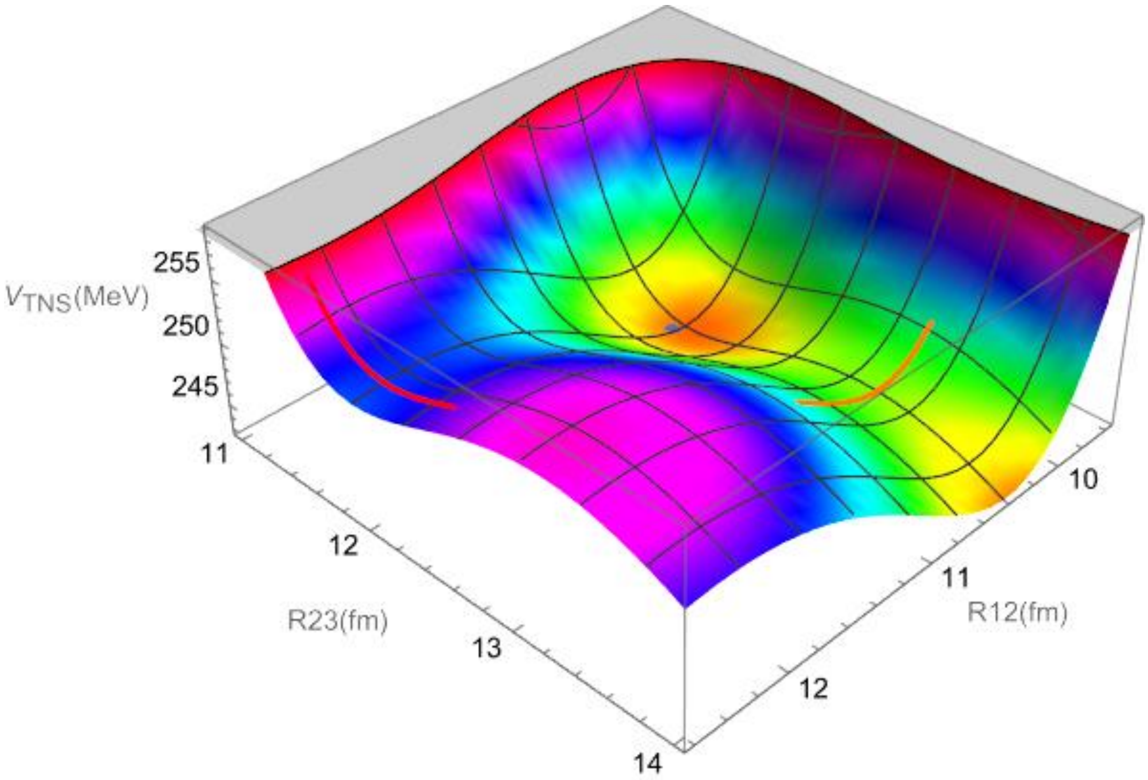

Figure 9.

In this case, barrier height B_R in DNS $^{50}Ca+^{70}Ni$ ($^{70}Ni+^{132}Sn$) is 15.054 MeV (7.11227 MeV). The same barriers in the case of TNS are 9.21 MeV and 3.34 MeV, respectively. This shows that TNS is less stable than corresponding DNSs.

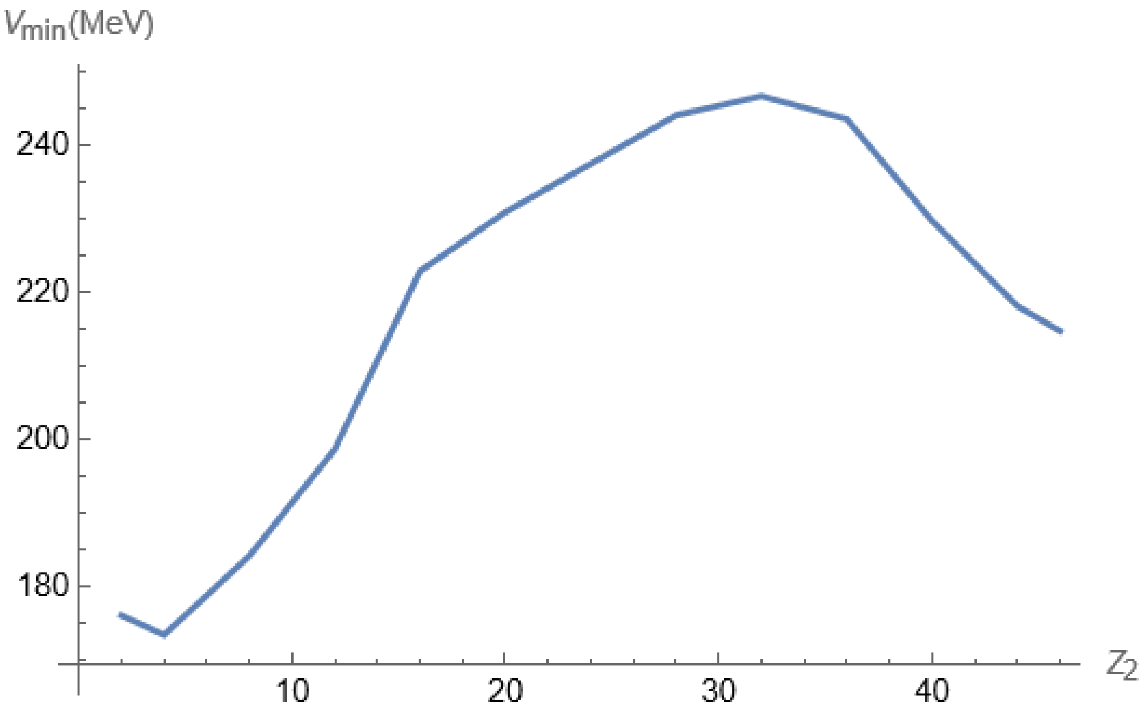

Figure 10.

Evolution of minimum of TNS interaction energy in spontaneous ternary fission of ^{252}Cf when right border nucleus is fixed to ^{132}Sn as a function of charge number of middle nucleus is given in Figure 10. We can see that by decreasing of the charge number of middle nucleus interaction energy firstly increases, reaching maximal values at $Z_2 = 32$ thereafter it decreases reaching the minimum value $Z_2 = 4$.

CONCLUSIONS

The potential energies of DNS and TNS formed in spontaneous fission of ^{252}Cf are calculated. Calculations show that decay barriers are strongly decreased in TNS in comparison to those in DNS. The minimum of interaction energy corresponds to TNS configurations when middle nucleus has smaller size.

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