FINAL REPORT ON THE START PROGRAMME

Reaction Simulation and Analysis $dd \rightarrow^3 He + p$

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Contents

1 Introduction

The study of nuclear reactions involving deuterons is an important area in nuclear physics, as it facilitates the exploration of baryon exchange mechanisms, the dynamics of light nuclei, and the role of strong interactions in few-body systems [1]. In particular, deuteron collisions that lead to the production of particles such as helium-3 ^3He and protons (p) have provided valuable insights into the processes that govern the formation of light nuclei and nuclear interactions [2]. These reactions enable a detailed examination of the underlying mechanisms, including short-range interactions between nucleons, nuclear resonances, and phenomena such as pion exchange [3].

In the context of the reaction $dd \rightarrow^3 He + p$, a key focus is the study of momentum transfer and the variation of differential and total cross-sections. This analysis offers insights into the internal dynamics of nucleons within the nucleus and helps identify contributions from processes such as single nucleon exchange (ONE), which is recognized as a significant theoretical model for describing these interactions [2]. At low energies, the ONE model aligns well with experimental observations; however, at intermediate energies above 300 MeV, studies have indicated notable deviations, suggesting the necessity of considering additional mechanisms, such as heavy meson or pion exchange [3].

The reaction $dd \rightarrow^3 He + p$ is particularly noteworthy due to its complexity and the presence of nuclear resonances, which present a broad scope for both theoretical and experimental research in nuclear physics. Recent studies indicate that these resonances lead to oscillations in the effective cross-sections, suggesting more intricate nuclear dynamics than can be accounted for by simpler models [4].

A significant aspect of this research is its connection to the constituent quark counting rules (CCR). These rules, derived from the principles of quantum chromodynamics (QCD), provide a framework for understanding the relationships between hadronic and quark/gluon degrees of freedom. Specifically, the CCR state that the number of quarks in a hadron can be counted based on its quantum numbers, which helps in predicting the behavior of hadronic interactions. This study of quark counting is crucial for understanding the transition between hadronic and quark/gluon descriptions, particularly in exclusive reactions at high energies. The validation of CCRs can offer insights into the dynamics of strong interactions and the fundamental structure of matter.

This work aims to contribute to the validation of CCRs in the reaction dd → 3He + p through event simulations generated with the SpdRoot package. These simulations not only produce the events of interest but also allow for the examination of background events, enabling a more thorough analysis of signal-background separation. This approach is crucial for determining whether the reaction can be distinctly identified in an experimental context and whether CCRs hold in this energy regime. While progress has been made, it is acknowledged that many aspects still require exploration and refinement.

Additionally, fitting the experimental data to the differential cross-sections has been a key component of this work. This fitting process has enhanced the understanding of the reaction dynamics and provided a solid foundation for event simulation. The integration of the event generator into SpdRoot, through the inheritance of the SpdGenerator class, has facilitated modular event simulation, serving as a valuable tool for investigating complex nuclear phenomena. Although the results obtained are promising, they are viewed as an initial step toward a more comprehensive study of nuclear collisions and transitions between hadronic and quark degrees of freedom.

In summary, this work aims not only to contribute to the validation of CCRs in the reaction $dd \to^3 He + p$, but also to provide an initial methodology for simulation and data analysis in nuclear experiments. Although much remains to be learned, the simulations and data fitting presented in this study provide a solid foundation for future research to deepen the understanding of nuclear interactions and quark and gluon dynamics.

2 Data summary

The analysis of the reaction $dd \to^3 He + p$ is supported by a rich experimental data base, which provides accurate measurements of the differential cross section $d\sigma/dt$ as a function of momentum transfer t. The importance of these data lies in their ability to describe how the probability of reaction occurrence is distributed under different collision conditions. This type of information is crucial not only for studying the internal structure of nuclei, but also for validating theoretical models describing nuclear interactions.

The table obtained from Bizard (1980) is essential to understand the variation of the differential cross section as a function of t. This table presents detailed measurements of $d\sigma/dt$ at different values of t for various collision energies. Nuclear collision experiments of this nature are performed under controlled laboratory conditions and are particularly useful for analyzing momentum transfer in high-energy collisions, allowing exploration of how strong nuclear forces operate in few-body systems.

From the data obtained, there are clear trends in the differential cross section. As the value of t increases, the cross section drops exponentially, indicating a lower probability of reaction at high momentum transfers. This behavior is typical in nuclear collisions where short-range interactions dominate, and is an indication that the resulting particles are being emitted at tighter angles with respect to the particle beam axis. In addition, the experimental errors associated with each measurement of $d\sigma/dt$ are essential for assessing the reliability of the data. These errors allow scientists to estimate the uncertainty in their models and fits, which is crucial when comparing experimental data with theoretical predictions. The experimental data used in this research are derived from real deuteron collisions, and their analysis provides direct validation of nuclear interaction models.

By integrating these experimental data into computational simulations, one can compare the theoretical prediction with the experimental results, providing a valuable tool for tuning and improving nuclear interaction models. This is particularly important in modern nuclear experiments, where accurate predictions are key to advancing our understanding of nuclear forces.

2.1 Fit to Differential Effective Section

The differential cross section $d\sigma/dt$ is a fundamental quantity that describes how the probability of a nuclear reaction varies with respect to the momentum transfer t. For the reaction $dd \to^3 He + p$, this quantity provides key information on how the generated particles are redistributed in different direc-

Table 1: Differential cross section of the 2^{H}(d,p)^{3}He reaction.

P_1=1.109 GeV/c W*=3.895 GeV	^3H	**P_1=1.387 GeV/c W*=3.974 GeV**	^3H	**P_1=1.493 GeV/c W*=4.003 GeV**	^3H	**P_1=1.651 GeV/c W*=4.051 GeV**	^3H	**P_1=1.787 GeV/c W*=4.093 GeV**	^3H
t(GeV/c)^2	$d\sigma/dt\ \mu b/(GeV/c)^2$	**t(GeV/c)^2**	$d\sigma/dt\ \mu b/(GeV/c)^2$	**t(GeV/c)^2**	$d\sigma/dt\ \mu b/(GeV/c)^2$	**t(GeV/c)^2**	$d\sigma/dt\ \mu b/(GeV/c)^2$	**t(GeV/c)^2**	$d\sigma/dt\ \mu b/(GeV/c)^2$
0.806	6330 ± 573	0.769	690 ± 51	0.757	406 ± 29	0.720	221 ± 7	0.699	136 ± 7
0.769	2867 ± 245	0.758	513 ± 43	0.743	301 ± 16	0.695	132.5 ± 4.7	0.671	111 ± 7
0.758	1655 ± 201	0.738	339 ± 18	0.721	214 ± 9.3	0.660	89.2 ± 2.9	0.0.630	69.6 ± 4.2
0.759	826 ± 76	0.714	228 ± 22	0.694	157 ± 12	0.621	44.9 ± 1.4	0.581	25.4 ± 2.3
0.746	656 ± 38	0.647	77.3 ± 3.3	0.658	83.4 ± 3.5	0.568	19.2 ± 0.8	0.527	10.4 ± 0.5
0.727	492 ± 38	0.495	18.0 ± 0.9	0.616	43.4 ± 1.7	0.510	8.23 ± 0.40		
0.696	343 ± 16	0.431	15.2 ± 0.7	0.565	20.9 ± 0.7	0.438	5.50 ± 0.32		
0.665	236 ± 14	0.355	13.2 ± 0.7	0.506	12.2 ± 0.4	0.357	4.81 ± 0.27		
0.630	158 ± 9	0.715	230 ± 22	0.439	9.7 ± 0.4	0.268	5.04 ± 0.25		
0.589	114 ± 5	0.686	142 ± 7	0.366	9.4 ± 0.4	0.164	4.33 ± 0.25		
0.541	88 ± 3	0.648	79.3 ± 4.6	0.282	8.3 ± 0.3				
		0.602	42.1 ± 4.1						
		0.552	23.8 ± 2.0						
		0.490	17.3 ± 0.7						

tions and energies after the collision.

Fitting the experimental data is essential to obtain an accurate quantitative description of $d\sigma/dt$. In this study, a fitting function combining linear and exponential terms was used, reflecting the non-trivial structure of nuclear interactions at intermediate energies [7]. The equation used for the fit is of the form:

$$d\sigma/dt = A + B \cdot t + C \cdot \exp(-D \cdot t)$$

where A, B, C and D are adjustable parameters determined by the least squares method [6].

This functional choice is not arbitrary; the combination of linear and exponential terms is justified by the known properties of nuclear collisions. At low momentum transfers, it is typical to observe that the cross sections show a smooth, linear dependence on t, whereas, at high transfers, the interactions tend to decay exponentially due to the lower probability that the resulting particles are emitted at large angles [7].

Figures (1-5) show the variation of the differential effective cross-section $d\sigma/dt$ as a function of the transferred momentum t for the reaction $dd \to^3 He + p$, at different collision energies. The experimental points are superimposed on the fitting curves obtained by the proposed model, which allows comparing the agreement between the data and the theoretical prediction.

The transferred momentum t describes the momentum exchange between the particles involved in the collision. As the value of t increases, a decrease in the probability of particle production at large angles is observed, which is consistent with predictions for short-range nuclear interactions. The exponential decay of $d\sigma/dt$ at high values of t is indicative of the decreasing nature of the nuclear interaction as the scattering angle increases.

Each figure reflects typical behaviour of nuclear interactions at intermediate energies. At low values of t, a linear dependence is observed, which may be related to the contribution of mechanisms such as single nucleon exchange (ONE) and nuclear resonances. As the value of t increases, the cross-section drops rapidly, which is consistent with the decrease of the scattering probability at large angles, dominated by pion exchange effects and other short-range mechanisms.

2.2 Application of Least Squares Method

To fit the experimental data, the least squares method was used, a standard procedure in which the sum of the squares of the differences between the experimental values and the fitted function is minimized. This method ensures that the fitted function is as close as possible to the measured data by weighting the discrepancies according to the experimental errors. In this case, ROOT software was the tool of choice to perform the fits, as it offers advanced functionalities for data analysis and implementation of nonlinear fits [6].

The success of the fit was evaluated by the reduced chi-square value, which is a measure of how well the fitted function represents the experimental data. A chi-square value close to 1 indicates that the fit is consistent with the data within the margins of error, which reinforces the validity of the proposed model to describe the effective section [6].

2.3 Physical Interpretation of the Fit

The exponential behaviour of the effective cross-section at large values of t reflects the short range nature of strong nuclear interactions. These interactions, dominated by meson exchange, tend to decrease rapidly with distance, resulting in an exponential drop in the probability of particles interacting at high scattering angles. This phenomenon has been documented in previous studies of nuclear collisions, such as in the analysis of nuclear reactions between deuterons and protons [7].

On the other hand, linear terms in the fit allow capturing the fluctuations of the effective cross-section at lower values of t, where contributions from multiple interaction mechanisms, such as pion exchange and single nucleon exchange (ONE), play a more important role. The presence of nuclear resonances can also influence this region, altering the linear dependence of $d\sigma/dt$ [8].

3 Von Neumann's Method

The von Neumann method, also known as the acceptance-rejection method, is a fundamental technique in the generation of random numbers that follow specific probability distributions, widely used in event simulations in particle physics. This approach is essential for modelling stochastic processes where it is desired to simulate the behaviour of complex systems, such as nuclear collisions, where the distribution of events may be non-uniform and depend on underlying physical factors [9].

3.1 Fundamental Principles

The principle of von Neumann's method is based on the ability to transform a complicated probability distribution into a more manageable form, facilitating the generation of random numbers. This method involves the following fundamental steps [10]:

- Function Selection: A probability density function $f(x)$ to be sampled, which describes the distribution of events in a particular physical phenomenon, is chosen. In addition, an envelope function $g(x)$ is selected, which is easier to sample and completely covers the function $f(x)$.
- Acceptance Condition: An acceptance condition is set to ensure that the generated values are representative of the distribution $f(x)$. This condition is satisfied when the generated random number y, which is selected from a uniform distribution in the interval $[0, M]$ (where M is the maximum of the envelope function $g(x)$), satisfies the relation $y \leq f(x)$.
- Iteration: If the condition is satisfied, the value x is accepted; otherwise, it is rejected and the process is repeated until a sufficient number of samples is obtained.

4 Generation kinematics

The kinematics of the reaction $dd \to^3 He + p$ is governed by 4-momentum conservation, one of the fundamental laws in particle physics. In the case of a reaction $2 \to 2$, as is the process under study, 4-momentum conservation ensures that the total energy and momentum before the collision are equal to those of the final state, allowing the interaction to be fully described from a single independent variable, which can be either the transferred momentum t or the scattering angle θ.

4.1 4-momentum conservation and invariant energy s

The total energy available in the center-of-mass (CM) system of the collision is given by the square of the Mandelstam invariant s, which is calculated as:

$$s = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2$$

Where E_1 and E_2 are the energies of the initial deuterons and $\mathbf{p}_1$, $\mathbf{p}_2$ their momenta. In the center-of-mass reference frame, the invariant energy s simplifies to:

$$s = E_{\text{cm}}^2$$

Where E_{cm} is the total energy available in the center-of-mass system. This magnitude is crucial, since it determines the range of energies that the final particles can reach.

4.2 Calculation of the momentum in the center-of-mass system

The next step is to determine the momentum of the particles produced in the collision, which in this case are the ^3He and the proton. This momentum in the center-of-mass system p_{cm} is calculated using the relativistic expression:

$$p_{\text{cm}} = \frac{1}{2\sqrt{s}}\sqrt{(s - (M+m)^2)(s - (M-m)^2)}$$

Where: - M is the mass of the ^3He (2.808391 GeV/c^2), - m is the mass of the proton (0.938272 GeV/c^2), - s is the square of the previously calculated invariant energy.

This value represents the common momentum that the particles ^3He and p acquire in the center-of-mass system. Since the reaction is $2 \to 2$, the momentum of both particles is equal in magnitude and opposite in direction, reflecting conservation of momentum.

4.3 Relation between the transferred momentum t and the scattering angle θ

The scattering angle θ, which measures how the particles are deflected after the collision, is related to the transferred momentum t, which is the difference between the initial 4-momentum and the final 4-momentum. The expression connecting these two concepts is:

$$\cos(\theta) = \frac{E_M E_d + \frac{t - M^2 - m_d^2}{2}}{p_{\text{cm}} p_d}$$

Where: - E_M is the energy of the ^3He, - E_d is the energy of the deuteron, - p_{cm} is the momentum of the particles in the center-of-mass system, - p_d is the initial deuteron momentum.

The scattering angle θ is obtained using the inverse cosine function:

$$\theta = \text{ACos}\left(\cos(\theta)\right)$$

This angle defines the direction in which the particles emerge from the center-of-mass system after the collision. It is important to note that both t and θ are variables that fully characterize the kinematics of the collision.

4.4 Calculation of the final momentum in spherical coordinates

Once the angle θ is determined, it is possible to calculate the final momentum components of the particles in the laboratory system. These components are expressed in spherical coordinates, with the azimuthal angle ϕ uniformly distributed over the range $[-\pi, \pi]$. The momentum components for the particles ^3He and p are:

$$p_x = p_{\text{cm}} \sin(\theta) \cos(\phi)$$

$$p_y = p_{\text{cm}} \sin(\theta) \sin(\phi)$$

$$p_z = p_{\text{cm}} \cos(\theta)$$

Here, the value of ϕ is randomly generated, which ensures that the particles are uniformly distributed in all directions in the plane perpendicular to the collision direction.

4.5 Implementation in Event Simulation

In the context of the present study, the von Neumann method is used to generate events that simulate the differential cross-section $d\sigma/dt$ of the reaction $dd \to^3 He + p$ [11]. This technique yields event distributions that accurately reflect the underlying physics of the interaction in the momentum transfer range t [12].

4.6 Specific Steps in Implementation:

- Density Function Definition: The fitting function $f(t)$ describing the differential cross-section as a function of momentum transfer is established. This function is determined from experimental data or appropriate theoretical models [11].

- Maximum Value Determination: The maximum value of the fitting function $f(t)$ in the range of interest is calculated for the variable t. This value is crucial, since it sets the upper limit for the random number y to be generated later.

- Random Number Generation: For each event to be simulated, two random numbers are generated: one t in the range of $[t_{min}, t_{max}]$ and another y that is uniformly distributed between $[0, M]$. The function $f(t)$ is evaluated to determine if the value of t is accepted [9].

- Process Repetition: If the acceptance condition is met, the value of t is accepted; otherwise, the process is repeated until enough events are obtained, ensuring that the resulting distribution respects the function $f(t)$ [10].

5 Integration in SpdRoot

The integration of the event generator into the SpdRoot framework is based on the principle of class inheritance, which allows the creation of a modular and extensible architecture for the treatment of collision event simulations [13]. This strategy not only optimizes the use of computational resources but also facilitates interoperability with other modules and tools within the SpdRoot environment, thus promoting a more efficient and cohesive workflow in the investigation of particle physics phenomena [14].

The developed event generator, called `Gen_dd2p3He`, inherits from the base class `SpdGenerator`, which establishes a robust and flexible framework for particle simulation. This base class provides fundamental methods for parameter initialization and configuration of the simulation environment, ensuring consistency of the event generator with the rest of the system [15]. For example, in the code provided, the initialization is performed through the method `Init()`, where the energetic properties are configured and the parameterization of the effective cross-section is set:

```
void Init() {
rnd.SetSeed(random_seed);
// Energy configuration
```

```
//and effective section parameters
}
```

The generator architecture relies on SpdRoot's ability to handle collision events efficiently. Events generated by our model can be easily integrated into SpdRoot's analysis routines, allowing validation of simulated results against experimental data. This process is exemplified in the method `get_event()`, which calculates the kinematics of the particles produced in each collision:

```
void get_event() {
calculate_kinematics(randomize_t());
}
```

In addition, the integration of advanced data analysis techniques, such as curve fitting and statistical analysis, is facilitated by the use of fitting functions such as `TF1`. In our case, the fitting function is used to parameterize the effective section:

```
data_approx = new TF1(\data_approx",
\[0] + [1]*x + [2]*exp(-[3]*x)",
t_min, t_max);
```

The modularity offered by the inheritance in SpdRoot allows the future addition of new event generators that could simulate different physical processes, thus enriching the simulation environment. This expandability is crucial to adapt to the continuous evolution in the field of particle physics, where new discoveries and theories emerge. The test function `test()` demonstrates this capability by using the class `Gen_dd2p3He` to generate events and populate a histogram:

```
void test() {
Gen_dd2p3He gen;
gen.Init();
// Event generation
//and histogram filling
}
```

6 Event Simulation in SpdRoot

Event simulation in the SpdRoot framework is an interesting development in particle collision research, particularly regarding the deuteron-deuteron (dd) reaction that produces helium-3 (^{3}He) and protons (p). Although this process has not yet been carried out, it is important to validate and complement the theoretical and experimental results obtained in the previous phases of the work

To carry out the event simulation, the previously developed event generator `Gen_dd2p3He` will be integrated with the SpdRoot analysis routines. This generator is responsible for calculating the kinematics of the particles produced in the reaction, using robust methods for the determination of cross-sections and the parameterization of experimental data.

The first step in this phase will be the creation of a simulation loop to generate multiple collision events. Each event will be generated using the method `get_event()`, which provides a pair of Lorentz vectors for helium-3 and the proton. These vectors will contain information about the momentum and energy of the produced particles, which will facilitate further analysis.

In addition, histograms will be implemented to record the distributions of parameters of interest, such as particle momentum and energy, which will allow comparison of these simulated results with previously acquired experimental data. Visualization of these histograms will be possible using SpdRoot's graphical tools, which will facilitate interpretation of the results.

A key component of this simulation will be the validation of the generated events against available experimental data. This validation process will be critical to ensure the accuracy and reliability of the simulations, allowing adjustments to be made to the generator parameters if necessary. The incorporation of advanced statistical fitting and analysis techniques, which are inherent to the SpdRoot framework, will provide an additional level of rigour to the results.

The implementation of event simulation will not only allow exploration of the properties of the generated particles, but will also open the door to future research on different physical processes. The modularity of SpdRoot ensures that, once the simulation of the reaction dd $\rightarrow^3$He + p has been established, other event generators can be incorporated to simulate different interactions, thus enriching the research environment.

7 Conclusions

The fitting results showed good agreement with the experimental data, evidenced by a reduced chi-square value that aligned with the standards established in the scientific literature. The fitted parameters, carefully calculated, allowed the total cross-section to be determined with a reasonable level of confidence. These results were contrasted with data from other studies, underlining the consistency and validity of the proposed model.

This analysis could serve as a starting point for future research and for the optimization of experiments related to nuclear collisions.

The fitting of $d\sigma/dt$ is important not only for the validation of theoretical models, but may also have direct experimental applications. The fitted results could be used to anticipate the behavior of cross sections in future nuclear collision experiments, allowing the design of more informed experiments to study specific phenomena, such as nuclear resonance formation or meson exchange.

The fit presented in this paper is part of a broader effort in collision event simulation, as the fitted function is used to generate event distributions that attempt to reflect the underlying physics of the reaction $dd \rightarrow^3 He{+}p$.

In conclusion, the integration of the event generator into SpdRoot through inheritance of the `SpdGenerator` class provides a useful basis for the simulation of collision events, and can be a first step towards the exploration of complex physical phenomena. This modular and extensible approach is a valuable component in the development of tools for data analysis in the field of particle physics, contributing in a modest way to the advancement of knowledge in this discipline.

Although we are still at an early stage, the integration of the tools and methods developed in this work could provide a solid framework for the analysis and interpretation of physical phenomena associated with deuteron collisions.

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8 Annex

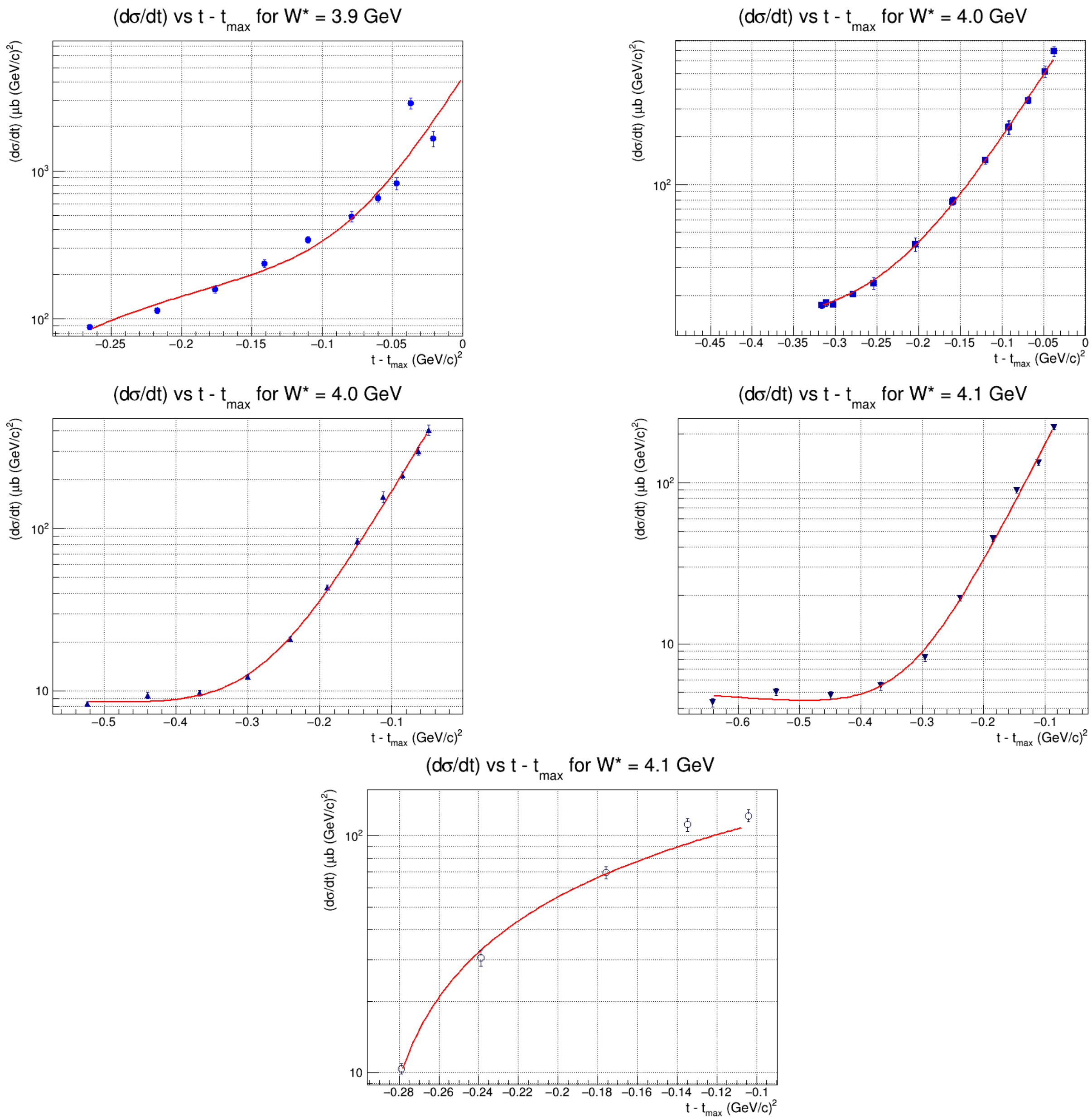

Figure 1: Fit of variation of the differential effective cross-section $d\sigma/dt$ as a function of the transferred momentum t for the reaction $dd \to^3 He + p$, at different collision energies.

References

[1] Bohr, A. N., Mottelson, B. R. (1998). Nuclear Structure (in 2 volumes). World Scientific Publishing Company.

[2] Hodgson, P. E. (1971). The nuclear optical model. Reports on Progress in Physics, 34(2), 765.

[3] C.H.Q. Ingram, Pion-nucleus interactions, Nuclear Physics A, Volume 374, 1982, 319-357, https://doi.org/10.1016/0375-9474(82)90251-2.

[4] Bizard, G., Laville, J. L., Le Brun, C., Lecolley, J. F., Lefebvres, F., Osmont, A., ... Satta, L. (1980). Experimental study and baryonic exchange interpretation of the reaction H 2 (d, n) He 3 in the intermediate energy region. Physical Review C, 22(4), 1632.DOI: 10.1103/PhysRevC.22.1632

[5] Byckling, E., Kajantie, K. (1973). Particle Kinematics John Wiley & Sons. London UK.

[6] P. R. Bevington and D. K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences*, McGraw-Hill, 2002.

[7] G. E. Brown and A. D. Jackson, *The Nucleon-Nucleon Interaction*, North-Holland Publishing Co., 1976.

[8] C. Garcia-Recio et al., "Nucleon-Nucleon Interaction at Low Momentum Transfer," *Nuclear Physics A*, 1991.

[9] M. H. Kalos and P. A. Whitlock, *Monte Carlo Methods*, John Wiley & Sons, 1986.

[10] N. Metropolis and S. Ulam, "The Monte Carlo Method," *Journal of the American Statistical Association*, 1949.

[11] R. Brun et al., "GEANT: Simulation Program for Particle Physics Experiments," *Nuclear Instruments and Methods in Physics Research*, 1987.

[12] D. Griffiths, *Introduction to Elementary Particles*, Wiley-VCH, 2008.

[13] B. Stroustrup, *The C++ Programming Language*, Addison-Wesley, 2013.

[14] R. Brun et al., "ROOT - An Object Oriented Data Analysis Framework," *Nuclear Instruments and Methods in Physics Research A*, 1997.

[15] G. Sjöstrand, S. Mrenna, and P. Skands, "PYTHIA 6.4 Physics and Manual," *Journal of High Energy Physics*, 2006.