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FINAL REPORT ON THE START PROGRAMME

Independent component analysis for linear optics restoration at NICA booster

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Abstract

In the operating conditions of the NICA accelerator complex, there is a need to perform measurements of linear optics of synchrotrons. There is a set of methods for linear optics obtaining based on measurement results. First group uses turn-by-turn orbit data detected by beam position monitors. The work consists in considering the motion of a particle beam and applying the method of analyzing independent components to data from an optical accelerator model to calculate the betatron tunes, beta functions and dispersions. The results of the work will allow us to evaluate the effectiveness of the method and the accuracy of calculating the parameters.

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Introduction

The NICA (Nuclotron based Ion Collider fAcility) project aims to create an accelerator complex for conducting research in the field of particle physics at JINR in a previously inaccessible area of experimental parameters and condition. Accelerator complex plays a pivotal role in contemporary research within the realms of elementary particle physics and nuclear physics.

Particle accelerators include electromagnetic elements that are used to change the direction of motion, focus and accelerate the flow of particles. Incorrectly set starting beam parameters lead to the appearance of oscillatory movements along the trajectory of motion. This leads to necessary of its orbit correction. Amplitudes of optical elements of the lines and correctors are used for the orbit control. Since the number of elements ranges from several dozen to several hundred, it is necessary to optimize the process of working with signals.

The objective of this study or investigation is the development mathematical methods, numerical algorithms for calculating main of parameters motion beam

In this work, the main focus is on the booster of the NICA accelerator complex. The key tasks of the work are as follows:

1) Generating beam position data taking into account the presence of noise and dumping relative to the ideal trajectory;

2) Using the method of independent components, we divide the multidimensional signal into additive subcomponents;

3) Analyze the data obtained.

1. Introduction to the System

1.1. The NICA accelerator complex

In 2009, work began at JINR on the design and construction of a new accelerator complex, NICA [1]. The goal of the NICA/MPD (Multi Purpose Detector) project is to create an accelerator complex designed to carry out an advanced particle physics research program at JINR.

The most important fundamental research directions in this field include:

- 1. Nature and Properties of Strong Interactions: Studying the strong interactions between the elementary constituents of the Standard Model of particle physics, namely quarks and gluons.
- 2. Phase Transition Search: Searching for signs of a phase transition between hadronic matter and QGP (Quark-Gluon Plasma), as well as exploring new states of baryonic matter.
- 3. Study of Strong Interaction and QGP Symmetry: Investigating the fundamental properties of strong interactions and QGP symmetry [3].

The complex will make it possible to accelerate and collide heavy ions, up to gold ions, in the optimal energy range, from the minimum – in the zone of extracted beams, to the most achievable energy in the center of mass system $\sqrt{\text{sNN}} = 11$ GeV/n (for Au⁺⁷⁹, in the nucleon-nucleon center of mass system) at the collider, with an average luminosity of L=10²⁷cm⁻² s⁻¹. Experiments will be carried out on fixed targets using Nuclotron beams at kinetic energy up to the maximum design energy (4.5 GeV/n). A Multi-Purpose Detector has been proposed for the Collider experiment. Another goal of the NICA project is to conduct experimental research on spin physics on oncoming polarized beams of protons and light nuclei [2].

The accelerator complex will provide beams of various particles with a wide range of parameters. This will allow for both applied and fundamental research in various fields of science and technology. Among them are the following [2]:

- Radiobiology and space medicine;
- Cancer therapy;
- Development of accelerator beam-controlled reactors and technologies for the transmutation of nuclear energy waste;
- Testing the radiation resistance of electronic devices.

Applied research using particle beams produced at the NICA complex is aimed at developing new technologies in materials science, solving environmental problems, creating new methods of energy production, cancer therapy, etc.



Figure 1 - Schematic view of the NICA accelerator complex [4]

NICA complex included [7]:

- The injection complex, consisting of two independent parts – for light and for heavy ions;

- The booster synchrotron (Booster) at magnetic rigidity of 25 T×m dedicated to acceleration of heavy ions;

- The upgraded Nuclotron, main accelerator of the NICA facility at magnetic rigidity of $38.5 \text{ T} \times \text{m}$;

- The NICA collider (two storage rings with two interaction points) at magnetic rigidity of 45 T×m, average luminosity $L = 10^{27}$ cm⁻² s⁻¹ for gold ion collisions at energy in the center of mass system \sqrt{sNN} up to 11 GeV/n;

- The experimental areas for applied research;

1.2. Booster design and systems

Superconducting booster synchrotron is the heavy ion injector of the Nuclotron. Main goals of the Booster operation are the following [7]:

- 1. Beam intensity at injection $(2 \cdot 10^9 \text{ ions of } ^{197}\text{Au}^{31+})$;
- 2. Acceleration at minimum loss by achievement of ultra-high vacuum conditions in the beam pipe;
- 3. Formation of the required beam phase volume by electron cooling application;
- 4. Acceleration of heavy ions to the energy required for effective stripping;

5. Fast extraction of the beam for injection into the Nuclotron.

A booster with a perimeter of 211 m and a structure of four periods is placed inside the yoke of a Synchrophasotron magnet. The maximum field of the Booster dipole magnets is 1.8 T (magnetic rigidity is 25 T×m), which corresponds to the $^{197}Au^{31+}$ ion energy of 578 MeV/n.



Figure 1 - Location of the main Booster systems [7].

Accumulation of gold ions and their acceleration to an energy of 578 MeV/ n, which is sufficient for subsequent stripping them to the state $^{197}Au^{79+}$. This makes it possible to significantly reduce the pressure requirements for the residual gas in the Nuclotron due to a decrease in the probability of ion recharge. At the same time, the final energy of the gold nuclei in the Nuclotron is 4.5 GeV/n. The use of electronic cooling in a Booster with an ion energy of 65 MeV/n will lead to a decrease in the longitudinal emittance of the beam to the value required for compression of the clot upon completion of its acceleration in the Booster.

1.3. Beam diagnostic system

The BPM (Beam Position Monitor) is a device designed to observe and measure the instantaneous position. These data are crucially important for the precise tuning of the accelerator and ensuring the stability of beam circulation, which directly impacts the quality and outcomes of experiments [6].

A beam position monitoring system typically consists of an array of sensors placed along the trajectory of the particle beam. These sensors detect the passage of charged particles, providing information about their positions at specific sections of the accelerator. Analyzing this data enables the determination of the beams spatial location, as well as its shape and dimensions.

A split pickup is an electrode between which a beam passes, and the electric field induced by it allows you to calculate the coordinate of the center of mass of the beam with respect to the coordinate system of the pickup electrodes. Either paired electrodes (Fig. 3(a), a) or a combination of signals from four electrodes (Fig. 3(a),

b) are used to calculate the beam coordinates.



Figure 3(a) - A split pickup with paired electrodes for measuring the position of the beam horizontally (a, 1) and vertically (a, 2) and a pickup in which the horizontal and vertical coordinates of the passing beam are calculated from signals from four electrodes (b).

The operation of the BPM is based on various principles of beam position measurement electromagnetic. The choice of a specific sensor types depends on the accelerators characteristics and the requirements placed on it.



Figure 3(b) - Working principle of the beam position monitor [6]

In the case of using diametrically opposite paired electrodes, the position of the beam in the simplest case is calculated by the formula:

$$x = \frac{a_x}{2} \frac{(U_1 - U_2)}{(U_1 + U_2)} = \frac{a_x}{2} \frac{\Delta X}{\sum X}$$

where a_x is the distance between the electrodes, U_1 and U_2 is the amplitude of the signals on the electrodes, ΔX and $\sum X$ is the so-called difference and sum signals from the electrodes.

BPM is usually located near each quadrupole magnet. Externally, it looks like any other element of the ring. The difference is only in the labeling.

Interpreting signals from sensors is a crucial step in the process of beams diagnostics. The acquired signals can be presented in analog or digital form and contain information about the distribution of particles within the beam in terms of coordinates, energies, and other characteristics. This ultimately enables the development of methods to enhance the quality of the beam.

In conclusion, the BPM stands as a pivotal instrument in ensuring the stable and high-precision operation of charged particle accelerators. Accurate information about the beam's position and its characteristics allows for the control and optimization of its movement, which is a critical requirement for successfully tuning the setup's design parameters and carrying out experimental and scientific research endeavors.

2. The theoretical part

2.1. The main parameters of the beam

When a particle moves within the lattice structure of an accelerator, a deviation in its transverse coordinate or momentum from the design value results in the emergence of betatron oscillations around the equilibrium orbit. The linear betatron oscillations of a particle in a cyclic accelerator are described by the Hill's equation in the accompanying coordinate system [9]:

$$\begin{cases} x'' + K_x(z)x = 0\\ y'' + K_y(z)y = 0 \end{cases}$$

where $x'' = \frac{d^2x}{dz^2}$, $y'' = \frac{d^2y}{dz^2}$, $K_x(z) = \frac{1}{B\rho} \frac{\partial B_y}{\partial x} + \frac{1}{\rho^2}$, $K_y(z) = -\frac{1}{B\rho} \frac{\partial B_y}{\partial x}$, B - magnetic field, ρ - radius of curvature of the trajectory. Here, *x* corresponds to the horizontal plane, and *y* corresponds to the vertical plane.

The solution, for instance, in the vertical plane, can be found in the form of two independent solutions [11]:

$$y_1(z) = e^{\frac{i\varphi z}{C}} p_1(z),$$
 $y_2(z) = e^{-\frac{i\varphi z}{C}} p_2,$

where φ is the accumulated horizontal φ_x or vertical φ_y betatron phase, which is determined by the following relation:

$$\cos\varphi = \frac{1}{2}traceM(z)$$

p1(z) and p2(z) are periodic functions of z, and M(z) is the transition matrix [11]:

$$p_i(z+C) = p_i(z), i = 1, 2, \dots N$$
$$M(z) = M(z + \frac{C}{z})$$

where *C* is the accelerator circumference.

The temporal variation of transverse coordinates and momentum is periodic but not sinusoidal. Therefore, the spectrum of their oscillations will contain a set of the betatron frequencies harmonics. The horizontal Q_x or vertical Q_y betatron frequency in a cyclic accelerator is defined as the number of betatron oscillation periods occurring within one turn:

$$Q = \frac{\varphi(z+C) - \varphi(z)}{2\pi} = \frac{1}{2\pi} \int_{z}^{z+C} \frac{dz}{\beta(z)}$$

where β is the corresponding beta-function [10].

Betatron frequencies are basic parameters of an accelerator, greatly influencing its effective operation. The stability of beam motion, luminosity in colliders, and the brightness of synchrotron radiation sources largely depend on the working point's position (its betatron frequencies) in the betatron resonance plane. Consequently, measuring betatron frequencies is among the primary diagnostic procedures during facility startup.

The integer part of the betatron frequency can be easily determined based on the number of periods of wave-like orbit distortion caused by local magnetic field perturbation.

The fractional part of the betatron frequency can be measured through spectral analysis of an array of coherent betatron oscillations, detected by a beam position monitor on each turn.

2.2 Independent component analysis

Independent component analysis (ICA) is a powerful blind source separation method. Compared to the Principal Component Analysis (PCA) ICA is more robust to noise, coupling, and nonlinearity [11, 12, 13]. The independent component analysis is applied to analyze simultaneous multiple turn-by-turn beam position monitor data of synchrotrons. The transverse motion of a beam in a Booster is composed of components driven by various physical factors. The sampled data are decomposed to physically independent source signals, such as betatron motion, synchrotron motion and other perturbation sources.

The essence of the method can be explained by the example of a cocktail party. Imagine that there are microphones that receive a signal, and several signal sources that produce independent signals. The task is to separate the mixture of signals. The ICA method considers turn-by-turn BPM data as linear mixtures of independent source signals. The data of position beam sampled by BPMs around the ring are put into a data matrix:

$$X = \begin{bmatrix} x_1(1) & \cdots & x_1(N) \\ \vdots & \ddots & \vdots \\ x_m(1) & \cdots & x_1(N) \end{bmatrix}$$

where *N* is the total number of turns, *m* is the number of BPMs. The element $X_i(j)$ is the reading of the i_{th} BPM on the j_{th} turn. This matrix can rewrite in other matrix:

$$X_{m \times n} = A_{M \times L} S_{M \times L}$$

The main task of ICA method consist in definition matrix S and matrix A.

$$C_s(\tau) = \langle s(t)s(t-\tau)^T \rangle$$

First, x is preprocessed to obtain mean-zero ($\bar{x} = \langle x \rangle$), whitened data ($zz^T = I$). This procedure called whiting. SVs from SVD of \bar{x} are separated via a cutoff condition λc (U₁, $\Lambda_1 \ge \lambda_c$), and $z = Y\bar{x} = \Lambda_1^{-1/2} U_1^T \bar{x}$. Then, $C_z(\tau_k)$ is calculated for a set of time lags (τ_k , k = 0, 1, ..., K). Since the modified time-lagged covariance matrix $\bar{C}_z(\tau_k) = (C_z(\tau_k) + C_z(\tau_k))^T / 2$ is real and symmetric, SVD is well defined:

$$\bar{C}_z(\tau_k) = W D_k W^T$$

where W is the unitary demixing matrix and D_k is a diagonal matrix. The Jacobi angle technique discussed in Ref. [15] is used to find the demixing matrix W, which is a joint diagonalizer for all $\bar{C}_z(\tau_k)$. A and s are calculate

$$A = Y^{-1}W$$
 and $S = W^T Y \bar{X}$

Each source signal s_i and its spatial distribution A_i , where A_i is the *i*-th column of A, is called a mode. The physical origin of a mode can be identified by its spatial and temporal functions. Temporal mode needed for analysis are contained in first ten columns. Matrix S includes important five temporal modes and when new conditions are added, the number may change. It is two pairs betatron and one synchrotron. The rest of the temporary modes are of no interest.

Fourier analysis modes gives frequencies of betatron oscillations for can using built-in math libraries. The betatron functions can be calculated as:

$$\beta_1 = c_1 (A_{b1,i}^2 + A_{b2,i}^2)$$
$$\beta_2 = c_2 (A_{b3,i}^2 + A_{b4,i}^2)$$

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The synchrotron mode is proportional to the variance:

$$D_x = c_3 A_{b5,i}$$

Where c_1 , c_2 , c_3 are scaling constant responsible for amplitude of betatron oscillations. A_{b1} and A_{b2} , A_{b3} and A_{b4} are pairs of betatron modes and A_{b5} is synchrotron mode.

In this work, the root mean square (RMS) error is used to estimate the accuracy of reproducing the parameters.

3. The practical part

3.1. Generating a dataset

At the first stage of the work, it is necessary to simulate the motion of an ion in a synchrotron without taking into account the coupling of transverse motions and longitudinal oscillations and taking into account these conditions.

For NICA booster calculation procedure was the following. Computer model of the accelerator is prepared with using code OptiMX [16]. The first model does not take into account the presence of coupling. The motion of an ion can be described by a matrix:

$$M_{1-2} = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}} \left(\cos(\mu_{1-2}) + \alpha_1 \sin(\mu_{1-2}) \right) & \sqrt{\beta_1 \beta_2} \sin(\mu_{1-2}) \\ -\frac{1}{\sqrt{\beta_1 \beta_2}} \left((1 + \alpha_1 \alpha_2) \sin(\mu_{1-2}) + (\alpha_1 - \alpha_2) \cos(\mu_{1-2}) \right) & \sqrt{\frac{\beta_1}{\beta_2}} \left(\cos(\mu_{1-2}) + \alpha_2 \sin(\mu_{1-2}) \right) \end{pmatrix}$$

Where μ_{1-2} – betatron phase advance.

The second model and all the calculations are made with taking into account coupling of transverse motions. From the model alpha-, beta-functions, betatron phase advances, and variables described value of coupling are exported. Based on this data circulating of an ion (center mass of the beam) in the ring is simulating. At each turn orbit position is saved for each BPM. The ion moving is calculated with using algorithm based on transformation eigen vectors of transfer matrices [16, 17]. Ion trajectory at turn n in phase-space in place where BPM m is located can be calculated as:

$$\begin{bmatrix} x_{n,m} \\ x_{n,m'} \\ y_{n,m} \\ y_{n,m'} \end{bmatrix} = Re(\sqrt{\varepsilon_1}(\bar{v}_1)_m e^{-i(\varphi_1 + (\mu_1)_m + n\mu_{1R})} + \sqrt{\varepsilon_2}(\bar{v}_2)_m e^{-i(\varphi_2 + (\mu_2)_m + n\mu_{2R})}) + \begin{bmatrix} Dx_m \\ 0 \\ D_m \\ 0 \end{bmatrix} (\frac{dp}{p})$$

where $\varepsilon_{1,2}$ - beam horizontal emittances, $\mu_{1,2}$ - betatron phase advances in two transverse planes, $\psi_{1,2}$ - initial phases of the ion, $\mu_{1R,2R}$ – betatron phase advances corresponding full turn in the ring, Dx, Dy - dispersion functions, dp/p - momentum deviation, n – turn number. Eigen vector \vec{v}_1 and \vec{v}_2 at azimuth where BPM m is installed are defined as:

$$\left(\vec{v}_{1}\right)_{m} = \begin{bmatrix} \sqrt{\left(\beta 1_{x}\right)}_{m} \\ -\frac{i\left(1-u\right)+\left(\alpha 1_{x}\right)_{m}}{\sqrt{\left(\beta 1_{y}\right)_{m}}e^{i\left(\nu_{1}\right)_{m}}} \\ \sqrt{\left(\beta 1_{y}\right)_{m}}e^{i\left(\nu_{1}\right)_{m}} \\ -\frac{iu+\left(\alpha 1_{y}\right)_{m}}{\sqrt{\left(\beta 1_{y}\right)_{m}}}e^{i\left(\nu_{1}\right)_{m}} \end{bmatrix}, \qquad \left(\vec{v}_{2}\right)_{m} = \begin{bmatrix} \sqrt{\left(\beta 2_{x}\right)}_{m}e^{i\left(\nu_{2}\right)_{m}} \\ -\frac{iu+\left(\alpha 2_{x}\right)_{m}}{\sqrt{\left(\beta 2_{x}\right)_{m}}}e^{i\left(\nu_{2}\right)_{m}} \\ \sqrt{\left(\beta 2_{y}\right)_{m}} \\ -\frac{i\left(1-u\right)+\left(\alpha 2_{y}\right)_{m}}{\sqrt{\left(\beta 2_{y}\right)_{m}}} \end{bmatrix}$$

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Generalized Twiss-functions α_{1_x} , β_{1_x} , α_{1_y} , β_{1_y} , α_{2_x} , β_{2_x} , α_{2_y} , β_{2_y} in (6) describe coupled transversal motion. In case of coupling absence non-zero values have only α_{1_x} , β_{1_x} and α_{2_y} , β_{2_y} , which go to commonly used alpha- and beta-functions of horizontal and vertical motions. v_1 and v_2 - relative phases of horizontal and vertical components corresponding eigen vectors. Constant *u* characterizes value of coupling [17].

In the mathematical environment of the Mathcad [21], to make calculations and fix the position of the beam on the BPM:



Figure 4 - Ion motion in horizontal phase-space without coupling (left) and with coupling (right)

In order for the data to be close to the real data from the accelerator, noise interference oscillation damping must be added. In this work, uses damping factor related to betatron tunes spread for Gaussian beam [22]:

$$F_{x} = \frac{1}{1+\theta^{2}} exp\left[-\frac{x_{k}^{2}}{2\sigma_{x}^{2}}\frac{\theta^{2}}{(1+\theta^{2})}\right]$$
(1)
$$\theta = 4\pi\Delta Qn; \quad \varphi_{x} = 2arctan\theta$$

Here x_k - initial amplitude of beam oscillations, σ - root-mean-square transversal size of the beam, ΔQ - spread of betatron tunes in the beam. An example of applying a coefficient to data:



Figure 5 - Data with damping.

Now it is necessary to prepare the data for further application of the ICA method, i.e. to decode the data. Next, I use the program [23] in Python language applicable to the ICA data. The data obtained will be analyzed in the following sections.

3.2. Data analysis without noise and damping

In order to test mathematical methods and evaluate the accuracy of restoring physical signals, the first calculations are performed on data without noise and damping of oscillation.



Figure 6 - First six temporal modes provided by ICA.

Components 1, 2 are betatron oscillations, components 4,5 are other betatron oscillations mode 3 – synchrotron motion. The other modes are noise. Fast Fourier analysis can calculate the betatron oscillation frequencies [19]:



Figure 7 - Frequencies of betatron oscillations

Results of restoration of betatron functions are also in a good agreement with Data from OptiMX [16]. Root-mean-square accuracy of betatron functions restoration reaches values for β_x and β_y are equal respectively 7×10^{-7} and 1×10^{-6} . Horizontal dispersion is restored with accuracy 6×10^{-4}



Figure 9 - Restored values horizontal dispersion

1×10⁴

These results help to confirm the accuracy of the ICA method. The data corresponds quite accurately to the target values (Fig. 9).

Ν

2×10⁴

3×10⁴

3.3. Data analysis with noise

0

To understand what effect noise can have on the system, to test the methods and evaluate the accuracy of restoring physical signals on data with noise, but without damping of oscillation.



Figure 10 - Frequencies of betatron oscillations

The figure (fig.10) shows the appearance of the effect of adding noise to the betatron frequencies. As the noise intensity increases, the spectrum becomes more noisy. In real systems, the noise can be so strong that it merges with the main frequency.



Figure 11 - ICA results for restoration of betatron functions

Also, the addition of noise caused an increase RMS of betatron functions restoration reaches values for β_x and β_y are equal respectively 4×10^{-5} and 9×10^{-5} . Horizontal dispersion is restored with accuracy 9×10^{-3} .



Figure12 - Restored values horizontal dispersion

3.4. Data analysis with damping

Damping has a great influence on the behavior of a real system. The law of the effect of damping is described above (1) and demonstrated in Figure 5. Consider the effect of adding damping on the result of the ICA method:



Figure 13 - Frequencies of betatron oscillations

When damping is added, the width of the main harmonic increases. The addition of damping caused an increase Root-mean-square accuracy of betatron functions restoration reaches values for β_x and β_y are equal respectively 0.045 and 0.054. Horizontal dispersion is restored with accuracy 9×10^{-3} .



Figure15 - Restored values horizontal dispersion

The effect of damping has a strong effect on the results of the ICA method, but results of restoration parameters are also in a enough good agreement with target data computed in the accelerator computer model.

3.4. Data analysis without noise and damping

Add noise and damping to the given beam positions. Such a system is the closest to the real system. To apply FT to data from Data analysis without noise and damping:



Figure 16 - Frequencies of betatron oscillations



Figure 17 - ICA results for restoration of betatron functions



Figure18 - Restored values horizontal dispersion

The width of the main harmonic spectrum of the betatron oscillation spectrum has become wider. Ehe value of the root-mean-square error of the restored values. For β_x , get 3×10^{-2} . For β_y , get 3×10^{-2} . The value of the root-mean-square error horizontal dispersion for is 3×10^{-1} .

4. Conclusion

The stages of the work at the JINR LHEP under the START summer program can be divided into two parts: theoretical and practical.

The theoretical part includes an excursion program on the LHEP, acquaintance with the goals and objectives of the NICA megaproject. By studying the structure of the Booster Accelerator, a detailed deepening into its component parts. The study of the beam motion in the accelerator, its main motion parameters, as well as mathematical methods of data processing. Methods for evaluating the linear optics of ring accelerators based on measurement results were also consider.

The practical part is the application of the Independent Component Method for processing data from the optical accelerator model. It is worth noting that this method has been successfully implemented in many other scientific accelerations centers. The main task was to test a method for calculating of betatron and synchrotron tunes, beta-functions and dispersions. The method was tested with different data that contained different levels of interference. For data that does not contain any interference, the method showed a recovery accuracy error of at least 10⁻⁵. The analysis of data containing noise and data containing damping showed an accuracy of 10⁻³ and 10⁻², respectively. The data that contained noise and Damping were the closest to real data, their analysis showed an error in the accuracy of restoration of 10⁻². The results obtained help to conclude that the method of independent components is acceptable.

5. Gratitude

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