JOINT INSTITUTE FOR NUCLEAR RESEARCH

Laboratory of Theoretical Physics

FINAL REPORT ON THE START PROGRAMME

Two-body problem

Supervisor:
Prof. Andrej Arbuzov

Student:
Azimzade Rustam, Azerbaijan
Moscow State University Baku Branch
Physics Faculty

Participation period:
July 17 - August 28, 2022

Dubna, 2022

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Andrej Arbuzov, Azimzade Rustam

02.08.2022

Abstract

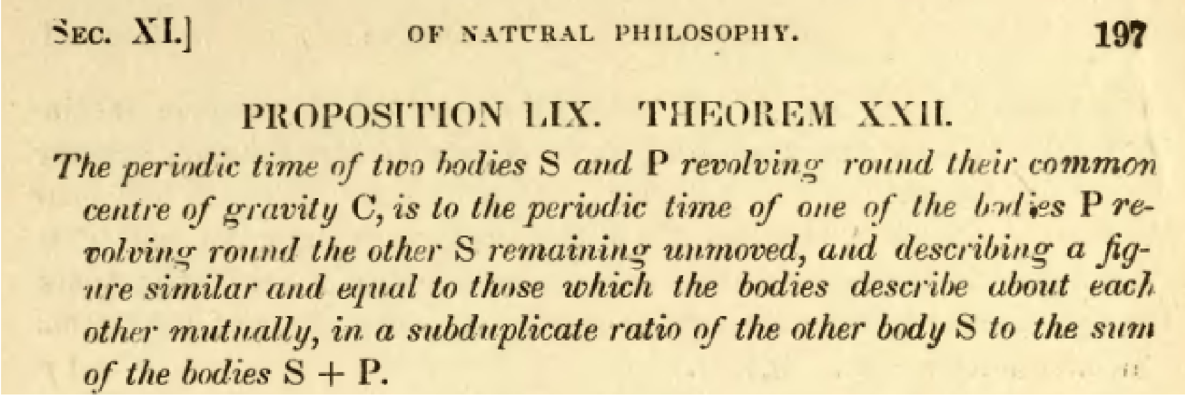
The main goal of the work for the period of summer practice was to study the problem of two bodies in various cases: in the classical mechanics, in non-relativistic and relativistic quantum mechanics. Methods for reducing the problem of two bodies to the case of the motion of one body in a static field were analyzed. A new method for reducing the relativistic problem of the interaction of two scalar particles proposed in the paper "On a novel equal time relativistic quasipotential equation for two scalar particles" [1] was analyzed in detail. We also studied the original general reduction method described in the preprint [2]. The proposed method was tested in detail for known solutions of the two-body problem, both in classical mechanics and in quantum mechanics.

1 Introduction

The problem of the two-body problem lies in the inability to always represent the interaction of two bodies with one motionless center of force: for example, the motionless Sun, around which the Earth moves. However, even this description is an approximation, since the Sun also moves under the influence of the Earth's gravity, although this movement is almost imperceptible. Moreover, there are systems in which interacting bodies have a comparable mass: binary stars, planets with satellites, and already in such systems it is necessary to take into account the motion of both bodies.

The most common method for solving the two-body problem is to reduce it to a problem describing the motion of one effective body in a central potential. A similar transition was even described by Newton for calculating the time of periodic rotation of two bodies around their common center of mass [3], see Fig. 1. There is also a more general approach in quantum field theory, the Bethe–Salpeter method [4]. Various methods are also used for numerically solving the problem of two or more bodies.

For the case of the motion of one body in the central potential, such solutions are already known in science as: the Kepler problem, the one-particle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PROPOSITION LIX. THEOREM XXII.

The periodic time of two bodies S *and* P *revolving round their common centre of gravity* C, *is to the periodic time of one of the bodies* P *revolving round the other* S *remaining unmoved, and describing a figure similar and equal to those which the bodies describe about each other mutually, in a subduplicate ratio of the other body* S *to the sum of the bodies* S + P.

Figure 1: Newton's quote.

Schrödinger equation, the quasipotential approach [5], the one-particle Dirac equation, the Klein-Fock-Gordon equation. Solutions are also known for the Droz–Vincent–Komar–Todorov equation [6-8] for scalar particles and the Dirac equation [9, 10] for spin particles. Nevertheless, if we consider the method of reducing the motion of a single body with a reduced mass to the problem, then it will be valid exclusively for the case of non-relativistic velocities of interacting particles. And in the case of relativistic velocities, the simple method of reduction to a one-particle problem no longer works due to the effects of relativistic interaction delay, etc. It is this problem that will be the basis for our further analysis of the corresponding two-particle equations, and the search for their new solutions in the case of two relativistic particles. This should make it possible to more accurately calculate the contributions of relativistic effects to the spectra of atoms, the experimental accuracy of which is constantly increasing.

In the first section of this work, we will consider the Kepler problem, and discuss its important consequences and methods regarding the logic of transition to different frames of reference: the C.M.S. (center of mass system), as well as the rest systems of the first and second bodies.

Then, the Schrödinger equation with a relativistic Hamiltonian will be constructed, and for the first time we will pay attention to the problem of ordering operators, which will not allow us to solve the problem directly. Therefore, in the next step, we will move to C.M.S. and see that then it is possible to find a solution for the wave function.

In the fourth Chapter, we will consider the Klein-Fock-Gordon equation, in which we will take into account the static Coulomb potential due to the lengthening of the time derivative, and also show that the magnetic interaction can be taken into account in the same way, lengthening the momentum.

In the final section, we will discuss the work done, as well as a further vector of development.

2 Kepler's problem

One of the tasks of practice was the study of problems of the motion of one body in the central potential (single-particle equations), to which the problem of two bodies is reduced.

Consider the case of a relativistic motion of a body in a central potential $U(\rho) = -\frac{\alpha}{\rho}$, where $\alpha = G_N m M$, G_N is the Newton constant.
As is known, the law of motion in the central field in quadratures is written by the following two equations:

$$t - t_0 = \pm \int_{\rho}^{\rho_0} \frac{d\rho}{\sqrt{\frac{2}{m}\left(E - U_{eff}(\rho)\right)}}. \tag{1}$$

$$\varphi - \varphi_0 = \pm \int_{\rho}^{\rho_0} \frac{p_\varphi d\rho}{m\rho^2 \sqrt{\frac{2}{m}\left(E - U_{eff}(\rho)\right)}}. \tag{2}$$

In the case of the Kepler problem

$$U_{eff}(\rho) = \overbrace{-\frac{\alpha}{\rho}}^{U(\rho)} + \frac{p_\varphi^2}{2m\rho^2}. \tag{3}$$

Therefore, in order to obtain a solution to the Kepler equation, we need to take the integrals (1) and (2) with such a potential. But first, for simplicity of taking the integral, we will make the following change of variables:

$$U = \frac{1}{\rho}. \tag{4}$$

$$dU = -\frac{1}{\rho^2} d\rho. \tag{5}$$

And then:

$$\varphi - \varphi_0 = \mp \int_{\rho}^{\rho_0} \frac{dU}{\sqrt{\frac{2m}{p_\varphi^2}\left(E + \alpha U - \frac{p_\varphi^2}{2m} U^2\right)}}. \tag{6}$$

Let's open the brackets and rewrite the expression in the following form:

$$\varphi - \varphi_0 = \mp \int_{\rho}^{\rho_0} \frac{dU}{\sqrt{\frac{2mE}{p_\varphi^2} + \frac{2m\alpha}{p_\varphi^2} U - U^2}}. \tag{7}$$

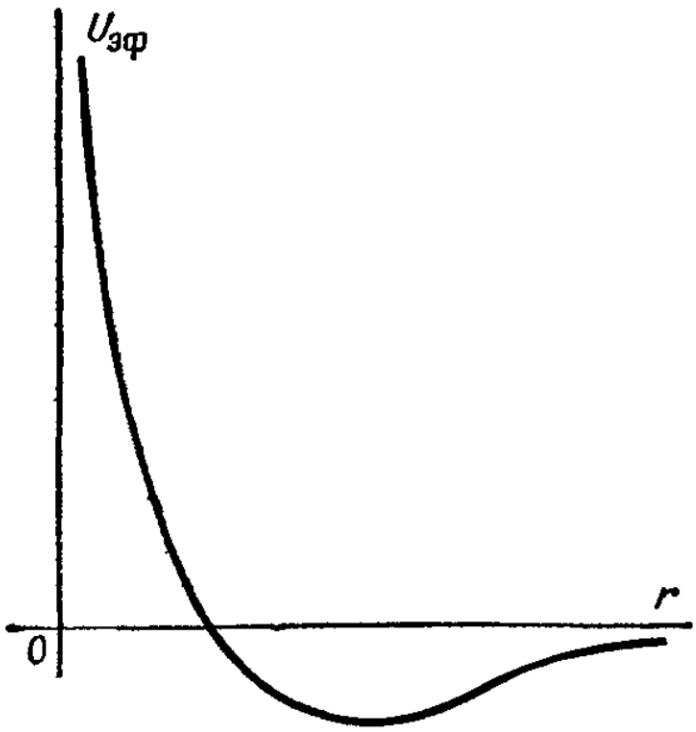

Figure 2: The effective potential energy versus radius.

Let's select a full square under the root:

$$\varphi - \varphi_0 = \mp \int\limits_{\rho}^{\rho_0} \frac{dU}{\sqrt{-\left(U - \frac{m\alpha}{p_\varphi^2}\right)^2 + \frac{m^2\alpha^2}{p_\varphi^4} + \frac{2mE}{p_\varphi^2}}}. \tag{8}$$

Now the integral is tabular, take it, and substitute the value for U in the answer

$$\varphi - \varphi_0 = \arcsin \frac{\frac{1}{\rho} - \frac{m\alpha}{p_\varphi^2}}{\sqrt{\frac{m^2\alpha^2}{p_\varphi^4} - \frac{2mE}{p_\varphi^2}}}\Bigg|_{\rho_0}^{\rho}. \tag{9}$$

$$\underbrace{\varphi - \varphi_0 + \arcsin \frac{\frac{1}{\rho_0} - \frac{m\alpha}{p_\varphi^2}}{\sqrt{\frac{m^2\alpha^2}{p_\varphi^4} - \frac{2mE}{p_\varphi^2}}}}_{\varphi_1} = \arcsin \frac{\frac{1}{\rho} - \frac{m\alpha}{p_\varphi^2}}{\sqrt{\frac{m^2\alpha^2}{p_\varphi^4} - \frac{2mE}{p_\varphi^2}}}. \tag{10}$$

$$\varphi - \varphi_1 = \arcsin \frac{\frac{1}{\rho} - \frac{m\alpha}{p_\varphi^2}}{\sqrt{\frac{m^2\alpha^2}{p_\varphi^4} - \frac{2mE}{p_\varphi^2}}}. \tag{11}$$

Multiply both sides of the equation by sin, and express the value ρ

$$\rho = \frac{\frac{p_\varphi^2}{m\alpha}}{1 + \sqrt{1 + \frac{2Ep_\varphi^2}{m\alpha^2}} \sin\left(\varphi - \varphi_1\right)}. \tag{12}$$

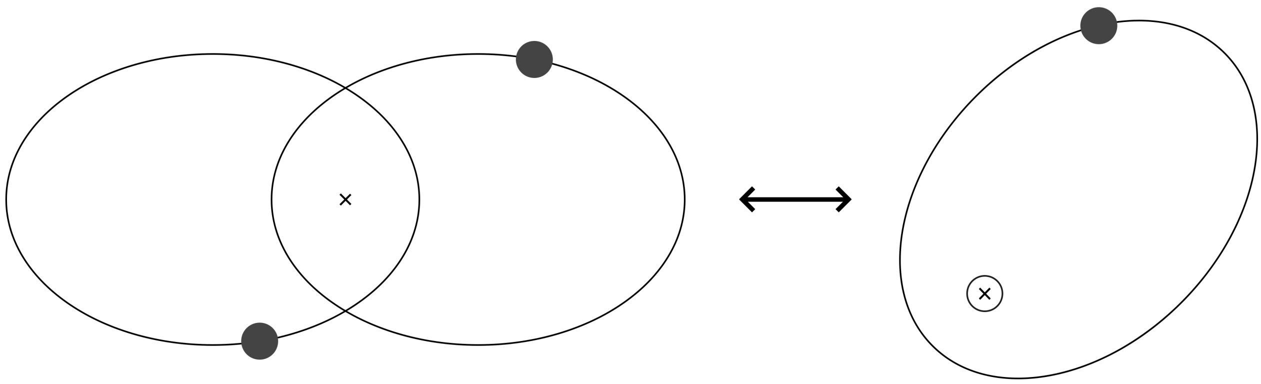

Figure 3: The motion of two bodies of equal mass around their common center of mass and bodies with a mass much less than the second

Since in our problem the coordinate φ is not explicitly included in the Lagrange function, the generalized momentum corresponding to it is the integral of motion. In our problem, the generalized momentum p_φ coincides with the moment M [10].

$$p_\varphi = M. \tag{13}$$

Therefore, taking this into account in the following notation, we have:

$$p = \frac{M^2}{m\alpha}; \qquad \varepsilon = \sqrt{1 + \frac{2EM^2}{m\alpha^2}}. \tag{14}$$

$$\rho = \frac{p}{1 + \varepsilon \sin(\varphi - \varphi_1)}. \tag{15}$$

Let $\varphi_1 = -\frac{\pi}{2}$, then:

$$\rho = \frac{p}{1 + \varepsilon \cos\varphi}. \tag{16}$$

We have obtained the equation of the conic section

$$\frac{M^2}{mr} = 1 + \sqrt{1 + \frac{2EM^2}{m\alpha^2}} \cos\varphi. \tag{17}$$

Now, if we want to go in this equation to the rest frame of the first particle, then the equation will take the following form:

$$\frac{M_2^2}{m_2 r} = 1 + \sqrt{1 + \frac{2E_2 M_2^2}{m_2\alpha^2}} \cos\varphi. \tag{18}$$

Below are some calculations that allow you to go to C.M.S., if you use the corresponding values the desired quantities in equation (17).

$$E_{C.M.S.} = \frac{m_1 V_1^2}{2} + \frac{m_2 V_2^2}{2} = \frac{p^2}{2m_1} + \frac{p^2}{2m_2} = \frac{1}{2}\left(\frac{(m_2 + m_1)\,p^2}{m_1 m_2}\right) = \frac{p^2}{2\mu}. \tag{19}$$

Consider the radius vector C.M.S, which is generally written as follows:

$$\vec{R} = \frac{\sum\limits_{n=1}^{N} m_\alpha \vec{r}_\alpha}{\sum\limits_{n=1}^{N} m_\alpha}. \tag{20}$$

Since we are considering the two-body problem, then $N = 2$

$$\vec{R} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1 + m_2}. \tag{21}$$

$$\vec{\rho} = \vec{r_1} + \vec{r_2}. \tag{22}$$

Let the origin of the coordinate system be at the point of the center of mass, then $|\vec{R}| = 0, |\vec{r_1}| = r_1, |\vec{r_2}| = -r_2$. So, we obtain the following system of equations:

$$\begin{cases} m_1 r_1 - m_2 r_2 = 0 \\ \rho = r_1 + r_2 \end{cases} \tag{23}$$

From here we find:

$$r_1 = \frac{m_2\rho}{m_1 + m_2}, \qquad r_2 = \frac{m_1\rho}{m_1 + m_2}. \tag{24}$$

The full list of the necessary quantities that allow you to switch to different reference systems is given below in the form of a table:

Ref. system	$\vec{p_1} = 0$	$\vec{p_2} = 0$	c.m.s.
Momenta	$\vec{p_2} = m_2\frac{\vec{p}}{\mu}$	$\vec{p_1} = m_1\frac{\vec{p}}{\mu}$	$\vec{p_1} = -\vec{p_2}, \lvert\vec{p_i}\rvert = p$
Velocities	$\vec{V_1} = 0, \vec{V_2} = \frac{\vec{p}}{\mu}$	$\vec{V_1} = \frac{\vec{p}}{\mu}, \vec{V_2} = 0$	$\vec{V_1} = \frac{\vec{p}}{m_1}, \vec{V_2} = -\frac{\vec{p}}{m_2}$
Angular momenta	$M = m_2\rho\frac{p}{\mu}$	$M = m_1\rho\frac{p}{\mu}$	$M = p\rho$
Energies	$E = \frac{m_2 p^2}{2\mu^2}$	$E = \frac{m_1 p^2}{2\mu^2}$	$E = \frac{p^2}{2\mu}$

It should also be noted that all the calculations that were done for transitions to different reference systems — in fact, are Galilean transformations.

3 Schrödinger equation

The Schrödinger equation has the form

$$\hat{H}\Psi(\vec{r}) = E\Psi(\vec{r}). \tag{25}$$

For the two-particle case, the Hamiltonian will be the sum of the energies:

$$\left(-\Delta_1\frac{1}{2m_1}-\Delta_2\frac{1}{2m_2}-\frac{\alpha}{r}\right)\Psi\left(\vec{r_1},\vec{r_2}\right)=E\Psi\left(\vec{r_1},\vec{r_2}\right). \tag{26}$$

This equation can be obtained as a non-relativistic limit of the following equation:

$$\left(\sqrt{\vec{p_1}^2+m_1^2}+\sqrt{\vec{p_2}^2+m_2^2}\right)\Psi\left(\vec{r_1},\vec{r_2}\right)=\frac{1}{i}\frac{\partial}{\partial t}\Psi\left(\vec{r_1},\vec{r_2}\right). \tag{27}$$

$$\sqrt{\vec{p}^2+m^2}=m\left(1+\frac{\vec{p}^2}{m^2}\right)^{1/2}\approx m+\frac{\vec{p}^2}{2m}-\frac{\vec{p}^4}{8m^3}+... \tag{28}$$

Note that the quoted relativistic equation doesn't include interactions, which can be added in a certain way.

Now, if we try to square this expression, it won't help us much, because we also run into the problem of operator ordering. Therefore, we will switch to the center of mass system in order to reduce this two-particle equation to a similar one-particle one.

$$\{\vec{p_1},\vec{p_2},\vec{r_1},\vec{r_2}\}\to\{\vec{P},\vec{p},\vec{R},\vec{r}\}. \tag{29}$$

where

$$\vec{r}=\vec{r_1}-\vec{r_2};\quad \vec{P}=\vec{p_1}+\vec{p_2};\quad \vec{R}=\frac{m_1\vec{r_1}+m_2\vec{r_2}}{m_1+m_2};\quad \vec{p}=\frac{\vec{p_1}m_2-\vec{p_2}m_1}{m_1+m_2}. \tag{30}$$

From these equalities, it is important for us to single out the following system of equations:

$$\begin{cases}\vec{P}=\vec{p_1}+\vec{p_2}\\ \vec{p}=\frac{\vec{p_1}m_2-\vec{p_2}m_1}{m_1+m_2}\end{cases} \tag{31}$$

From which we get:

$$\begin{gathered}\vec{p_1}=\vec{P}\frac{m_1}{m_2}+\vec{p}\\ \vec{p_2}=\vec{P}\left(\frac{m_2-m_1}{m_2}\right)-\vec{p}\end{gathered} \tag{32}$$

Let us now return to the general form of the Schrödinger equation:

$$\left(\frac{\vec{p_1}^2}{2m_1}+\frac{\vec{p_2}^2}{2m_2}-\frac{\alpha}{r}\right)\Psi\left(\vec{r_1},\vec{r_2}\right)=E\Psi\left(\vec{r_1},\vec{r_2}\right) \tag{33}$$

We also find what the energy in C.M.S. is equal to

$$\dot{\vec{r}}=\vec{v}=\frac{\vec{p_1}'}{m_1}-\frac{\vec{p_2}'}{m_2}. \tag{34}$$

$$\dot{\vec{r_1}} = \left(-\frac{m_2\dot{\vec{r}}}{m_1+m_2}\right) = \vec{v_1} = m_1\vec{p_1}; \quad \dot{\vec{r_2}} = \left(-\frac{m_1\dot{\vec{r}}}{m_1+m_2}\right) = \vec{v_2} = m_2\vec{p_2}. \tag{35}$$

$$E = E_1 + E_2 = \frac{m_1\vec{v_1}^2}{2} + \frac{m_2\vec{v_2}^2}{2} = \frac{m_1}{2}\left(-\frac{m_2\dot{\vec{r}}}{m_1+m_2}\right)^2 + \frac{m_2}{2}\left(-\frac{m_1\dot{\vec{r}}}{m_1+m_2}\right)^2 =$$
$$= \frac{\left(m_1m_2^2 + m_2m_1^2\right)\dot{\vec{r}}^2}{2\left(m_1+m_2\right)^2} = \frac{m_1m_2\dot{\vec{r}}^2}{2\left(m_1+m_2\right)} = \frac{\mu\dot{\vec{r}}^2}{2}. \tag{36}$$

$$\frac{\vec{p_1}^2}{2m_1} + \frac{\vec{p_2}^2}{2m_2} - \frac{\alpha}{r} \xrightarrow{\textit{transition to the C.M.S.}} \frac{\vec{P}^2}{2\left(m_1+m_2\right)} + \frac{\vec{p}^2}{2\mu} - \frac{\alpha}{r}. \tag{37}$$

Denoting $M = m_1 + m_2$, we have

$$\left(\frac{\vec{P}^2}{2M} + \frac{\vec{p}^2}{2\mu} - \frac{\alpha}{r}\right)\Psi\left(\vec{r_1},\vec{r_2}\right) = E\Psi\left(\vec{r_1},\vec{r_2}\right). \tag{38}$$

We represent the wave function as a product:

$$\Psi\left(\vec{r_1},\vec{r_2}\right) = \Psi_R\left(\vec{R}\right)\Psi_r\left(\vec{r}\right). \tag{39}$$

Further, we will look for a solution only for $\Psi_R\left(\vec{R}\right)$

$$\frac{\vec{P}^2}{2M}\Psi_R\left(\vec{R}\right) = -E_R\Psi_R\left(\vec{R}\right). \tag{40}$$

$$\frac{\partial^2/\partial\vec{R}^2}{2M}\Psi_R\left(\vec{R}\right) = -E_R\Psi_R\left(\vec{R}\right). \tag{41}$$

$$\frac{\partial^2}{\partial\vec{R}^2}\Psi_R\left(\vec{R}\right) = -2ME_R\Psi_R\left(\vec{R}\right). \tag{42}$$

We will look for a solution in the form $\Psi_R\left(\vec{R}\right) = e^{\lambda R}$

$$\frac{\partial^2}{\partial\vec{R}^2}e^{\lambda R} = -2ME_Re^{\lambda R}. \tag{43}$$

$$\lambda^2e^{\lambda R} = -2ME_Re^{\lambda R}. \tag{44}$$

$$\lambda = i\sqrt{2ME_R}. \tag{45}$$

Substitute this value into the solution and get:

$$\Psi_R\left(\vec{R}\right) = e^{i\sqrt{2ME_R}R}. \tag{46}$$

4 The Klein-Fock-Gordon equation

The Klein-Fock-Gordon equation is a relativistic analogue of the Schrodinger equation, it looks like this:

$$\frac{1}{c^2}\partial_0^2\Psi - \partial_x^2\Psi - \partial_y^2\Psi - \partial_z^2\Psi + \frac{m^2c^2}{\hbar^2}\Psi = 0. \tag{47}$$

Where

$$\partial_0 = \frac{\partial}{\partial t}; \qquad \partial_x = \frac{\partial}{\partial x}; \qquad \partial_y = \frac{\partial}{\partial y}; \qquad \partial_z = \frac{\partial}{\partial z}. \tag{48}$$

This entry uses the following Minkowski metric:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{49}$$

$$\left(\frac{1}{c^2}\partial_0^2 - \partial_x^2 - \partial_y^2 - \partial_z^2 + \frac{m^2c^2}{\hbar^2}\right)\Psi\left(t, \vec{r}\right) = 0. \tag{50}$$

If we consider that $\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$ — the Laplace operator, the record can be simplified to the following form:

$$\left(\frac{1}{c^2}\partial_0^2 - \Delta + \frac{m^2c^2}{\hbar^2}\right)\Psi\left(t, \vec{r}\right) = 0. \tag{51}$$

For even greater simplicity, we will take the values of the speed of light and Planck's constant equal to unity ($c = \hbar = 1$). They can always be restored later in the desired formula in the only possible way from the dimension.

$$\left(\partial_0^2 - \Delta + m^2\right)\Psi\left(t, \vec{r}\right) = 0. \tag{52}$$

To take into account the static Coulomb potential in the equation, we make the following substitution:

$$\partial_0 \to \partial_0 - \iota\frac{\alpha}{r}. \tag{53}$$

And for the magnetic interaction:

$$p_\mu \to p_\mu - eA_\mu. \tag{54}$$

$$A_\mu \to \left\{A^0; \vec{A}\right\}. \tag{55}$$

Where

$$\alpha = \frac{e^2}{4\pi}; \qquad A^0 = -\frac{e}{4\pi r}; \qquad \vec{A} - \frac{e\vec{v}}{4\pi r}. \tag{56}$$

$$\left(\left(\partial_0 - \iota\frac{\alpha}{r}\right)^2 - \Delta + m^2\right)\Psi\left(t, \vec{r}\right) = 0. \tag{57}$$

$$\left(\partial_0^2 - 2\iota\partial_0\frac{\alpha}{r} - \frac{\alpha^2}{r^2} - \Delta + m^2\right)\Psi\left(t, \vec{r}\right) = 0. \tag{58}$$

Given that $-\iota\partial_0\Psi = E\Psi$ we get:

$$\left(E^2 - 2E\frac{\alpha}{r} + \frac{\alpha^2}{r^2} + \Delta - m^2\right)\Psi\left(t, \vec{r}\right) = 0. \tag{59}$$

$$\left[\Delta + \left(E - \frac{\alpha}{r}\right)^2 - m^2\right]\Psi\left(t, \vec{r}\right) = 0. \tag{60}$$

In order to obtain the eigenfunctions of this equation, we represent the wave function as the product of its harmonic and the corresponding radial part:

$$\Psi\left(\vec{r}\right) = R_l\left(\vec{r}\right)Y_{lm}\left(\theta, \varphi\right). \tag{61}$$

For convenience, we also introduce the values:

$$n_r = n - k, \qquad k = j + \frac{1}{2}, \qquad \gamma = \sqrt{k^2 - \alpha^2}. \tag{62}$$

where n is the principal quantum number and j is the quantum number of the total-angular-momentum

Then we get an analytical solution for energy levels in the form:

$$E = \frac{m}{\sqrt{1 + \left(\frac{\alpha}{\gamma + n_r}\right)^2}}. \tag{63}$$

We introduce the Mandelstam invariant variable s, which is more convenient to describe the energy of our system:

$$s = E^2_{C.M.S.} = \left(p_1 + p_2\right)^2 \tag{64}$$

Here p_1 and p_2 - is four momenta of the first and second particles, respectively. Finally we get the spectrum of our two–particle bound state in the C.M.S.

$$\begin{gathered}E_{C.M.S.} = \sqrt{s} = \left[m_1^2 + m_2^2 + 2m_1E_2\right]^{1/2} = m_1 + m_2 - \\ - \frac{\alpha^2}{2n^2}\mu - \frac{\alpha^4}{2n^3}\mu\left(\frac{1}{k} - \frac{3}{4n} + \frac{m_1m_2}{4n\left(m_1 + m_2\right)^2}\right) - \frac{\alpha^6}{2n^3}\mu\times \\ \times\left[\frac{1}{4k^3} + \frac{3}{4nk^2} - \frac{3}{2n^2k} + \frac{5}{8n^3} + \frac{\mu}{\left(m_1 + m_2\right)}\left(\frac{1}{2n^2k} - \frac{3}{8n^3}\right) + \frac{\mu^2}{\left(m_1 + m_2\right)^2}\frac{1}{8n^3}\right] + ...\end{gathered} \tag{65}$$

5 Conclusions

In the process of working for the period of summer practice, solutions to the two-body problem were studied in detail, in particular, the Kepler problem, the Schrödinger equation with a relativistic Hamiltonian, the Klein-Fock-Gordon equation. Solutions of a series of problems on the motion of a single body (particle) in a central potential are analyzed in detail. The literature containing solutions of the problems under consideration has been studied. Separate results from this literature are independently reproduced in detail. The transformations of Galileo and Lorentz during transitions between reference systems in two-body problems are studied. The method of perturbation theory has been mastered in applications describing the spectra of bound states. And also mastered the method of lengthening the time derivative in the relativistic equations of Dirac and Klein-Fock-Gordon to take into account the Coulomb potential.

Also, one of the tasks of the practice was to study the method shown in the preprint [2], in which, using the Maupertuis's principle of least action, we can reduce the two-particle problem to the description of the motion of one particle in an external static potential, even at relativistic velocities. The Maupertuis's principle, in particular, states that inertial forces do not contribute to the action minimization problem. Therefore, it is possible to pass to the rest frame of one of the interacting particles and solve the problem of minimizing the action in this frame. The resulting answer, for example, for the value of the binding energy must then be converted by transition to the C.M.S. In this case, the fact that the two-body problem simultaneously has solutions in the scattering channel and in the bound state channel (in the cases of interest to us) is essentially used. Knowing that for the scattering channel, transformations between frames of reference are carried out according to Galileo's laws in the nonrelativistic case or the case of Lorentz transformations in relativistic kinematics, we can make the corresponding transformations for bound states as well.

In the near future, I plan to continue working on an article that will form the basis of my future thesis under the guidance of my supervisor for the summer practice — Andrej Borisovich Arbuzov.

Acknowledgements

In the end, I would like to thank: my supervisor — Andrej Borisovich Arbuzov for his active mentoring throughout our joint work, the leadership of the Joint institute for nuclear research for financial support; Elena Genadievna Karpova, who provided prompt feedback throughout the entire time from the moment the application was submitted, as well as my curator — Nazim Huseynov Ali-oglu, who created the opportunity for my arrival and assisted in the search for a supervisor.

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