JOINT INSTITUTE FOR NUCLEAR RESEARCH
Bogoliubov Laboratory of Theoretical Physics

FINAL REPORT ON THE SUMMER STUDENT PROGRAM

Quark interaction in classical and non-relativistic cases

Supervisor:
Dr. Dmitri Igorevich Kazakov

Student:
Ivanovski Viacheslav, Russia
Moscow Institute of Physics and Technology

Participation period:
July 01 – August 12

Dubna, 2018

Abstract

Quark interaction is studied from classical equations of Yang-Mills theory. Solving these equations the characteristics of field and energy of quark interaction is determined. It is shown that there are not any properties of quantum picture of interaction in classical level. In addition, interaction of quark and antiquark through effective Cornell potential in non-relativistic limit is considered and masses of $c\bar{c}$ and $b\bar{b}$ mesons are calculated.

1 Introduction

It is believed that quark and antiquark that form meson are confined by a force that is caused by potential that rises with the distance. It is assumed that this interaction occurs due to gluon field that compresses the force into a thin tube between particles. Everywhere inside of this tube there is no gluon field.

It is interesting to test the extent to which details of this picture are confirmed by properties of exact solution in classical Yang-Mills theory with gauge group $SU(3)$. In this paper classical solution for this problem is obtained.

The properties of solution strongly contradict the picture above.There are no hints of the growth of the potential with the distance or compression of the field in a tube. Moreover, static quarks interact by a force of classical Coulomb type, due to which they are attracted.

To solve these contradictions and properly describe properties of mesons way was proposed. Non-relativistic quantum-mechanical problem of quark interaction was considered. It was assumed that they attract due to effective potential – Coulomb plus linear part. Calculating the energies of bounded states one may obtain the masses of mesons. This way works only for c and b quarks, because they are quite heavy so the non-relativistic approximation may be used.Furthermore, they have long lifetime to construct bounded states.

The paper is arranged as follows. Sec.2 introduces basic propositions of the classical Yang-Mills theory and system of particles interacting with this field, covers general properties of this interaction and treats solution of the Yang-Mills equations. Sec.3 is denoted to quantum-mechanical problem of interaction of two quarks and calculating masses of mesons. In sec. 4 auxiliary figures are presented.

2 Classical case

Quark interaction based on Yang-Mills vector field A_μ, with gauge group $SU(3)$. The form of field-strength tensor is:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu; A_\nu] \tag{1}$$

or componentwise:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{2}$$

We study the $SU(3)$ theory with coupling strength g and use component notation: A^a_μ, $a = \overline{1,8}$, and matrix notation $A_\mu = -igA^a_\mu \lambda_a/2$,where λ_a — Gell-Mann matrices. The commutator is in Lie algebra $su(3)$.

Yang-Mills equations of these object reads[1]:

$$\begin{aligned} D_\mu F_{\mu\nu} &= g j_\nu, \\ D_\mu &= \partial_\mu + g[A_\mu, \ ]. \end{aligned} \tag{3}$$

Electric and magnetic fields are derived from field-strength tensor:

$$\begin{aligned} E^a_i &= F^a_{0i}, \\ F^a_{ij} &= -e^{ijk} H^a_k. \end{aligned} \tag{4}$$

The energy for Yang-Mill field has form:

$$E = \frac{1}{2} \int dr \left(\mathbf{E^a E^a} + \mathbf{H^a H^a} \right). \tag{5}$$

To determine j_ν, a current of matter, one should turn to theory of quarks.

Quark can be represented as a triplet relatively to $SU(3)$:

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

u_1, u_2, u_3 are 4-component spinors. Components of u correspond to different colors of quark. Denote colors 1,2,3 as red green and blue, for instance green quark looks like:

$$u = \begin{pmatrix} 0 \\ u_2 \\ 0 \end{pmatrix}$$

Current of quark is written

$$j^\mu_a = -i\bar{u}\frac{\lambda_a}{2}\gamma^\mu u \tag{6}$$

In this section only the case of resting quarks is considered, therefore $j^m u$ is proportional to $\delta^{\mu 0}\delta(\mathbf{r}\text{-}\mathbf{a})$, where $\mathbf{a}$ is a location of the particle. Hence form for current of quarks of red, green and blue colors respectively:

$$\begin{aligned} j(r)^\mu_a &= \delta^{\mu 0}\delta(\mathbf{r}\text{-}\mathbf{a}) \left(\delta_{a3} + \frac{1}{\sqrt{3}}\delta_{a8} \right) \\ j(g)^\mu_a &= \delta^{\mu 0}\delta(\mathbf{r}\text{-}\mathbf{a}) \left(-\delta_{a3} + \frac{1}{\sqrt{3}}\delta_{a8} \right) \\ j(b)^\mu_a &= \delta^{\mu 0}\delta(\mathbf{r}\text{-}\mathbf{a}) \left(\frac{-2}{\sqrt{3}}\delta_{a8} \right) \end{aligned} \tag{7}$$

Consider interaction between red and antired quarks. Let red be at the point $\mathbf{a}$ and antired at the point $-\mathbf{a}$. Hence we obtain system (3) with the source:

$$j_a^\mu = \delta^{\mu 0}\delta(\mathbf{r}\text{-}\mathbf{a})\left(\delta_{a3} + \frac{1}{\sqrt{3}}\delta_{a8}\right) - \delta^{\mu 0}\delta(\mathbf{r}+\mathbf{a})\left(\delta_{a3} + \frac{1}{\sqrt{3}}\delta_{a8}\right) \tag{8}$$

Before solving this system one can obtain important result. Quark interaction energy has the form $1/a$.This is due to the fact that when a is changed, the solutions, as is not difficult to verify, undergo only scale transformations.

System (3) with source (8) is solved numerically, using scheme "cross"[3]. Results are shown on figure 1, it is $\mathbf{E^3}$ field lines. This solution consides with classic Coulomb case (figure 2). This conformity is in agreement with [4], where it is shown that if currents of different particles are commute, then system (3) has only the classical Coulomb solution.

3 Quantum case

Consider quark-antiquark system, which interact through potential:

$$V(r) = -\frac{A}{r} + B\frac{r}{a}, \tag{9}$$

where A, B are real positive constants, r is distance between particles, and a$=\frac{2\hbar^2}{mA}$ The reduced mass of this system equals $m/2$, where m is the mass of a quark. Substituting $V(r)$ into Schroedinger [2] equation one can obtain:

$$-\frac{\hbar^2}{m}\frac{d^2\Psi}{dr^2} + \left[-\frac{A}{r} + \frac{Br}{a} + \frac{l(l+1)}{r^2}\right]\Psi = E\Psi. \tag{10}$$

After substitution $x = r/a$ equation (10) takes the form:

$$\left(\frac{d^2}{dx^2} + \mathcal{E} - \left[-\frac{1}{x} + bx + \frac{l(l+1)}{x^2}\right]\right)\Psi = 0, \tag{11}$$

where $b = B/E_0$, $E_0 = \frac{mc^2}{4}(\frac{A}{\hbar c})$ and $E = \mathcal{E}E_0$. After changing variable such as $x = u^2$ and substituting $\Psi = \sqrt{u}g(u)$ one can obtain:

$$g''(u) + \left(4 - 4bu^4 + 4Eu^2 - \left[\frac{4l(l+1) + 3/4}{u^2}\right]\right)g(u) = 0 \tag{12}$$

Putting $z = (2\sqrt{b})^{1/3}$ and $g = z^{\alpha+1}e^{-\frac{z^3}{3}}f$ into 12, the equation below is yielded:

$$f'' = 2\left(z^2 - \frac{\alpha+1}{z}\right)f' + \left(2(\alpha+2)z - \beta z^2 - \gamma\right) \tag{13}$$

where $\gamma = \frac{4}{(2\sqrt{b})^{2/3}}$, $\beta = \frac{4\mathcal{E}}{(2\sqrt{b})^{4/3}}$ and $\alpha = 2l + \frac{1}{2}$. This equation is suitable for appling AIM (see appendix A and [5],[6])

To calculate meson masses following method is used. First the energy of bounded state is determined, then the quark and antiquark masses are added to it. Varying parameters A and B, one can obtain good estimation for meson masses. This approach is applied for $c\bar{c}$ mesons $(\psi(1S), \chi_{c0}(1P), \psi(2S), \chi_{c0}(2P))$ and $b\bar{b}$ mesons $(\Upsilon(1S), \Upsilon(2S), \Upsilon(3S), \Upsilon(4S), \chi_{b0}(1P), \chi_{b0}(2P), \chi_b(3P), \Upsilon(1D))$. Results are presented in tables 1 and 2. The best values of $\frac{A}{\hbar c}$ and $\frac{B}{E_0}$ for $c\bar{c}$ are 0.9 and 1.1, for $b\bar{b}$ are 0.14 and 1.9. Theoretical predictions match with the experimental data [7] well.

Table 1: Masses of $c\bar{c}$ mesons.

	$\psi(1S)$	$\psi(2S)$	$\chi_{c0}(1P)$	$\chi_{c0}(1P)$
Precise masses, GeV	3.096	3.686	3.414	3.918
Calculated masses, GeV	3.0	3.7	3.4	3.9

Table 2: Masses of $b\bar{b}$ mesons.

	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\Upsilon(4S)$
Precise masses, GeV	9.460	10.023	10.355	10.579
Calculated masses, GeV	9.4	9.9	10.4	10.6
	$\chi_{b0}(1P)$	$\chi_{b0}(2P)$	$\chi_b(3P)$	$\Upsilon(1D)$
Precise masses, GeV	9.912	10.255	10.534	10.163
Calculated masses, GeV	9.9	10.3	10.5	10.2

4 Figures

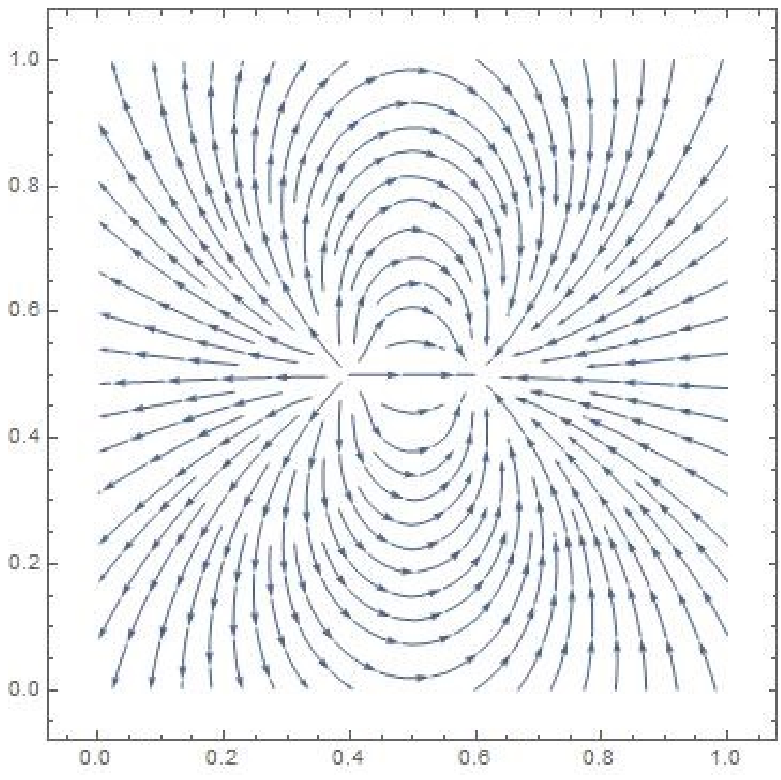

Figure 1: $\mathbf{E^3}$ field lines of quark interaction.

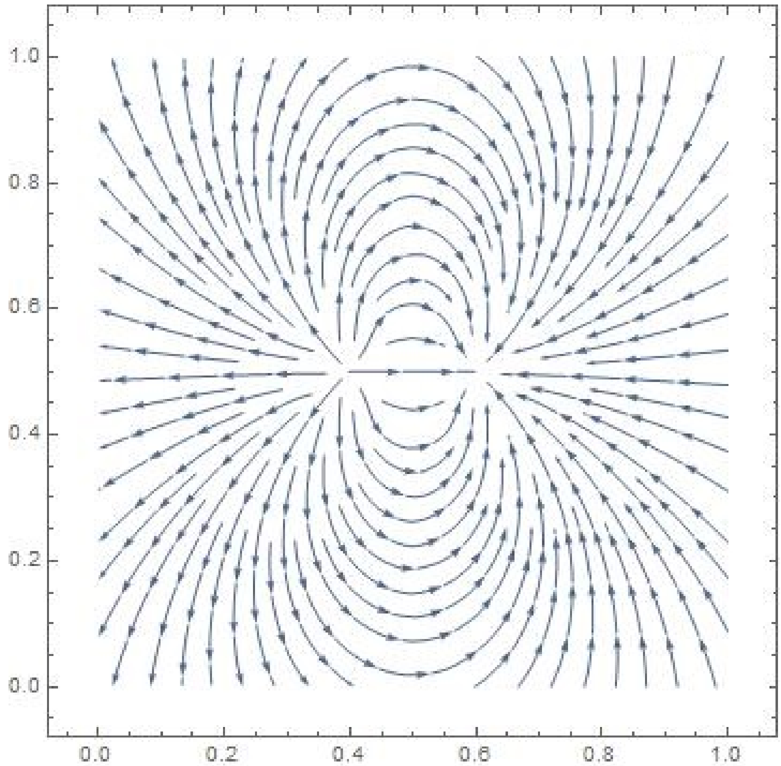

Figure 2: **E** field lines of charge interaction.

5 Acknowledgement

I would like to express my gratitude to my scientific supervisor D. I. Kazakov, BLTP staff, and JINR, provided financial support.

Appendix A

AIM is an asymptotic iteration method. It is used to solve second order linear differential equation such as:

$$y'' = \lambda_0 y' + s_0 y \tag{14}$$

where λ_0 and s_0 are C^∞ functions. AIM can be used for solving the Schroedinger equation. Using the functions:

$$\begin{aligned} \lambda_n &= \lambda_{n-1} + s_{n-1} + \lambda_0 \lambda_{n-1} \\ s_n &= s'_{n-1} + s_0 \lambda_{n-1} \end{aligned} \tag{15}$$

one can obtain energy levels through following condition for some large n:

$$\delta_n(x,E) = s_n(x,E)\lambda_{n-1}(x,E) - s_{n-1}(x,E)\lambda_n(x,E) = 0 \tag{16}$$

References

[1] Rubakov V. A. Classical gauge fields. Editorial URSS, 1999.

[2] Landau L. D., Lifshitz E. M. Quantum mechanics, non-relativistic theory. Science, 1989.

[3] Petrov I. B., Lobanov A. I. Lectures on computational mathematics. Binom, 2006.

[4] Rasidze O. Sources of nonlinear field. Dubna, 1984.

[5] Hakan Ciftci, H. F. Kisoglu. Schrodinger Equation for the Cornell Potential by the Asymptotic Iteration Method. 10.05.2017.

[6] H. Ciftci, R. L. Hall, N. Saad, J. Phys. A: Math. Gen. 36 (2003) 11807.

[7] The Review of Particle Physics (2018) M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).