Joint Institute for Nuclear Reaserch

Summer Student Program Final Report

Modelling of Rapidly Rotating Neutron Stars

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1 Abstract

The purpose of this report is to summarize the conclusions and data obtained by making numerical models of equilibrium sequences of rapidly rotating neutron stars in General Relativity. The evolutionary sequence models of these stars are solutions of Einstein's equations for the stationary gravitational field in axial symmetry. I determine important physical parameters for such stars as maxiumum-mass, central energy density and angular momentum which may provide relevant information for gravitational collapse into a blackhole. The rotating neutron star (RNS) code, used to construct these models, is written by Nikolaos Stergioulas in C language in 1993. In (2)Summary section, I provide a short theory of the topic and interpretation of the results. In (3)Methods section I describe the numerical methods used in the computing and conclusion of the results. In (4)Figures section of this report, I use 2 of the equations of states provided by the programmer to analyze the relations between the neutron star parameters. In (5)Acknowledgements section, I express my gratitude for all the people who were of assistance in the making of this report. In References section I outline the sources, books, articles I have used as help for the study.

2 Summary

In this report I present the results of a survey of relativistic rapidly rotating neutron stars (RNS) for tabulated equations of state (EoS). I will consider only uniformly rotating NS and in this sense, we can divide the NS evolutionary equilibrium sequences into two groups: stable and unstable. We say that such a sequence is stable only if there is a loss in angular momentum as the central energy evolves at rest mass. If there is no loss or if there is a positive gain, then the model is considered to be unstable.

Now we can also divide evolutionary sequences into normal sequences and supramassive sequences. The normal ones have static spherically symmetric solutions at one end and supramassive seq. contain no such solutions. The boundary between the two is the maximum-mass normal seq. which joins onto the maximum-mass static solution. The reason why the supramassive seq. are so interesting is that they exist because of relativistic effects and as the name suggests, their mass exceeds the normal mass of nonrotating/static NS, which will eventually result in catastrophic collapse into a blackhole.

There are 4 limits that should be considered when building a neutron star:

2.1 Static

- Where angular momentum and angular velocity converge to 0. These are simply the solutions to TOV eqs. for spherically symmetric models.

2.2 Mass-shed

- We reach this limit when the neutron star is rotating sufficiently rapidly that the gravitational attraction is not enough to keep matter bound to the surface.

2.3 Stability

- Where an equilibrium solution is marginally stable to quasi-radial perturbations. The stability limit begins at maximum-mass on static limit sequence and usually terminates near the maximum-mass point on the mass-shed limit sequence(maximum mass for rotating star). In other words, we find that there is a maximum mass limit after which collapse can occur.

2.4 Low-mass

- The limit under which the NS cannot form. We will not be considering this limit in this report.

2.5 Conclusion

In this report, we see that in order to be able to build realistic neutron stars, we need to put limits and constraints on various parameters. We also see that both rotating and static stars can be unstable after passing a specific upper limit in the mass. So after reaching a critical point in central energy density, the neutron star will certainly collapse and this critical point is measured on the basis of the specific EoS that are used, and its value is different for every EoS.

3 Methods

The method with which the RNS were analized was made possible by the code written by N. Stergiuolas. On successful running of the code, it prints out 17 physical parameters shown below.

ϵ_c central energy density

M gravitational mass

M_0 rest mass

R_e radius at the equator

Ω angular velocity

Ω_p angular velocity of a particle in circular orbit at the equator

T/W rotational/gravitational energy

$cJ/GM_\odot^2$ angular momentum

I moment of inertia

Φ_2 quadrupole moment

h_+ height from surface of last stable co-rotating circular orbit in equatorial plane

h_- height from surface of last stable counter-rotating circular orbit in equatorial plane

Z_p polar redshift

Z_b backward equatorial redshift

Z_f forward equatorial redshift

ω_c/Ω ratio of central value of potential ω to Ω

r_e coordinate equatorial radius

r_p/r_e axes ratio (polar to equatorial)

The following values for the physical constants are used: $c = 2.9979 \times 10^{10}$ cm/s, $G = 6.6732 \times 10^{-8} \mathrm{g}^{-1}\mathrm{cm}^3\mathrm{s}^2$, $m_B = 1.66 \times 10^{-24}$gr, and $M_\odot = 1.987 \times 10^{33}$gr.

The theory that the code is based on uses a metric in the following form:

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2) + e^{\gamma-\rho}r^2(d\phi - \omega dt)^2 \tag{3.1}$$

where the metric potentials $\rho, \gamma, \alpha, \omega$are functions of r and θ only.

The stress tensor has the following form:

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu}, \tag{3.2}$$

where ϵ is total energy density, P is pressure and u is 4-velocity.

The numerical methods used in the code are Runge-Kutta 4th Order and Interpolation. Should the reader like a more extensive reviews on these methods, they could check 3rd and 17th Chapter of Numerical recipes in C, S. Teukolsky, W. Press.

The code is programmed to work in two modes: first one is with tabulated equations of state and the other is with a polytropic equation. The tabulated one contains 4 columns: energy density (in g/cm^3), pressure (in dynes/cm^2), enthalpy (in cm^2/s^2), and baryon number density (in cm^{-3}).

RNS has several options which allow you to specify model parameters and choose from different output formats. A model is defined uniquely by specifying two parameters - one will always be the central energy density and the other can be one of the following: mass, rest mass, angular velocity, angular momentum, or the ratio of the polar coordinate radius to the coordinate equatorial radius.

The parameters are specified using the following flags:

-e central energy density in gr/cm^3

-r axes ratio

-m mass in $M_\odot$

-z rest mass in $M_\odot$

-o angular velocity in $10^4 s^{-1}$

-j angular momentum in $GM_\odot^2/c$

4 Figures

We will start with the FPS equation which is rather stiff.

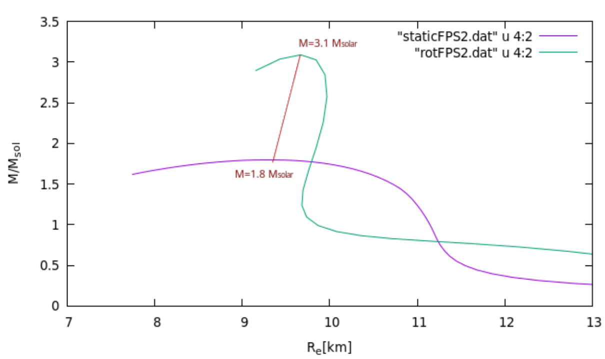

Fig.1 Mass-Radius relation. Static and rotating cases.

Fig.1 is a plot of Mass-Radius relation of the FPS equation of state. It should be read from right to left as the central energy density increases with the decrease of the equatorial radius. The line with the lesser maximum-mass is for the static neutron star, and the line with the higher maximum mass is for rotating neutron star(mass shed limit). The red line shows the stability limit between maximum mass of static stars and rotating stars. Left to that line the neutron star models are unstable and right to that line - stable, meaning that the left side is unstable to small radial perturbations, and we can see on the right side that even with slightly bigger radial perturbations, the change of mass is still positive, so the star will stay stable as long as it doesn't cross the red line (stability limit). The same conclusions can be drawn if we make a Mass/Central energy density plot.

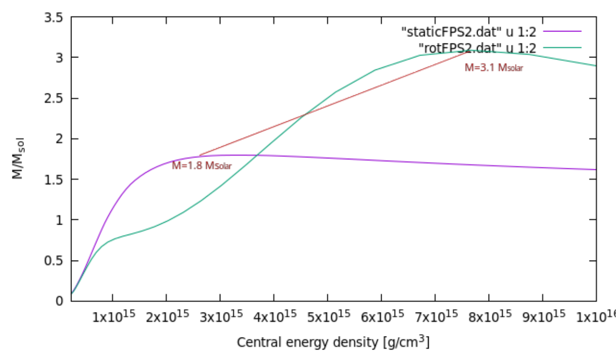

Fig.2 Mass-Central energy density relation. Static and rotating cases.

Fig.2 should be read from left to right. On this plot, again we see that the maximum mass limit for a static neutron star with FPS EoS is equal to 1.8 solar masses, and for a rotating star - 3.1 solar masses. Again we draw the red line as the stability limit and see that all neutron stars, this time right to the limit, are unstable, as the equatorial radius decreases counterproportionally to the central energy density. So we can conclude that:

$$M > 1.8 M_\odot \textit{ unstable static stars}$$

$$M > 3.1 M_\odot \textit{ unstable rotating stars}$$

The second equation of state we will be looking at will be F. Analogously, we make two plots in this case two.

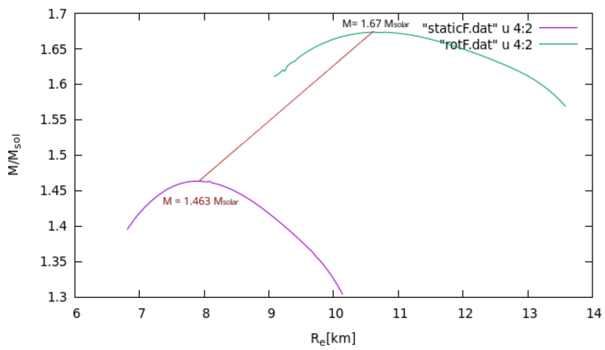

Fig.3 Mass-Radius relation. Static and rotating cases.

In this case, the maximum masses for static and rotating stars are lower. Again, the same relation can be seen when we draw the stability limit. On Fig.3, left from the limit are the stars that are unstable, and right from it - stable.

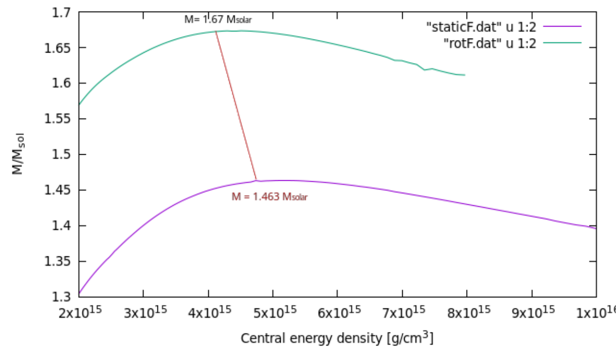

Fig.4 Mass-Central energy density relation. Static and rotating cases.

On Fig.4 we see the relation between the mass and central energy density for static and rotating stars with EoS F. After drawing the line for the stability limit, we can see that right from the limit, the stars are unstable, and left from it - stable. So we get:

$$M > 1.463 M_{\odot} \; \textit{unstable static stars}$$

$$M > 1.67 M_{\odot} \; \textit{unstable rotating stars}$$

5 Acknowledgements

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